THE PRICE OF ANARCHY IN AN UNCAPACITATED DETERMINISTIC SHIPMENT CONSOLIDATION GAME

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ABSTRACT

THE PRICE OF ANARCHY IN AN UNCAPACITATED DETERMINISTIC SHIPMENT CONSOLIDATION GAME

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Collaboration of companies for logistics activities result in more efficient use of logistics resources which leads to reduction of logistic costs. In this study, multiple shippers that make shipments from the same origin to the same destination are under consideration. The shipment requests of the shippers arrive over time at a deterministic uniform constant rate and the shippers consolidate their shipments on uncapacitated trucks. The trucks are dispatched at a certain frequency to maximize the profit gained from the shipments. Two settings are analyzed, where in the first shippers act as a coalition, and in the second, each of these shippers acts selfishly. The former is analyzed under a cooperative game while the latter under a non-cooperative Nash game. For the cooperative game the structure of the characteristic function is analyzed and the properties such as monotonicity, superadditivity, core existence and convexity are shown. For the non-cooperative game the conditions under which the Nash equilibrium exists are identified. Through numerical analysis the price of anarchy and the Gini coefficient are quantified by comparing the total profit and average individual profit of each shipper under cooperative and non-cooperative games.

Keywords: shipment consolidation, cooperation game, non-cooperation game, price of anarchy, inventory management,profit allocation

ÖZ

KAPASİTE KISITININ OLMADIĞI BİR SEVKİYAT KONSOLİDASYONU İŞBİRLİĞİ OYUNUNDA ANARŞİNİN BEDELİ

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Firmaların lojistik faaliyetler için iş birliği yapmaları, lojistik kaynakların daha verimli kullanılmasını ve lojistik maliyetlerinin azalmasını sağlamaktadır. Bu çalışmada aynı çıkış noktasından aynı varış noktasına sevkiyat yapan birden çok göndericiler ele alınmıştır. Gönderi talepleri zaman içinde deterministik tekdüze sabit bir oranda gelir ve gönderiler kapasite kısıtı olmayan araçlarda konsolide edilir. Göndericiler sevkiyatlardan elde edilen karı en üst düzeye çıkarmak için araçları belirli bir sıklıkta gönderir. Göndericiler bir koalisyon olarak hareket ettiği ve N göndericinin her birinin bencilce hareket ettiği iki senaryo analiz edilmiştir. Birincisi işbirliği oyunu altında, ikincisi işbirliği olmayan bir Nash oyunu altında analiz edilmiştir. İşbirliği oyunu için karakteristik fonksiyonun yapısı analiz edilmiştir ve monotonluk, üsttoplamlılık, çekirdek varlığı ve dışbükeylik gibi özellikler gösterilmiştir. İşbirliği olmayan oyun için Nash dengesinin var olduğu koşullar belirlenmiştir. Sayısal analiz yoluyla anarşinin bedeli ve Gini katsayısı işbirliği olan ve işbirliği olmayan oyunlarda toplam kârı ve her bir göndericinin ortalama kârını karşılaştırarak ölçülmüştür.

Anahtar Kelimeler: sevkiyat konsolidasyonu işbirliği, kooperatif oyun, kooperatif olmayan oyun, anarşinin bedeli, envanter yönetimi, kar tahsisi

This paper is dedicated to my family. I am thankful to my family for supporting me throughout my journey of this thesis study.

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CHAPTER 1

INTRODUCTION

For delivering finished goods or receiving raw materials, logistics operations are essential for companies. A transportation cost is incurred for each shipment of companies due to fuel consumption, carrier costs such as salary, insurance and truck costs of leasing or maintenance. Total transportation cost over a time period grows with the increase of transportation needs for shippers.

Today, there is an extensive growth of logistic activities globally. By 2024, the worth of global logistics market is expected to exceed 6.8 trillion euros (Statista, 2021). It shows that the logistics operations of companies are expanding and the transportation becomes a significant issue. So, the cost reduction activities have become greatly critical for companies. In this sufficiently large market, shippers with same objective may come together and try to operate efficiently for cost reduction. But, it seems that it was challenging for companies due to facing with fluctuating demand, restriction on due dates, financial problems or several obligations that companies need to obey and it results with low utilization of transportation assets which decreases profitability of the companies and also increase environmental damage. Saenz (2012) complains about the low utilization rates of freight vehicles in the European Union and suggests collaboration on transportation activities for increasing efficiency of the freight vehicles.

Collaboration is expressed working together with the objective of completing tasks and eventually achieving joint goals (Liao et al., 2017). In transportation, shippers can collaborate in several ways for their common benefit. They can make investments jointly on transportation assets and share these assets for reqular transportation operations. Also, they can collaborate by consolidating administrative operations by hiring co-workers or sharing technologies they developed for the ease of. Lastly, shippers may make collaboration by consolidating their requested shipments to increase the utilization of the transportation assets and, as a result of that, to decrease their transportation cost and/or to maximize profit made by shipments. An example for shipper collaboration can be given from Nistevo, Elogex, and Transplace where they manage a collaborative logistics network and use the Internet as a common computing platform for to provide the chance to track of their costs incurred. (Ergun et. al., 2017). Two companies of Nistevo network collaborate and make shipment schedules together and it results with a combined 19% savings for both shippers. In this study, we focus on a shipment consolidation case where shippers make joint shipments and they aim to maximize profit.

Transportation cost is not the only cost item that shippers need to handle. Shippers need to deal with waiting costs of goods in addition to the transportation costs. In shipment consolidation, shippers wait until the joint shipment to deliver their requested goods. A waiting cost of the goods are incurred for the waiting time of joint shipment based on the priority criteria of shippers of goods. If they want to reduce the shipment frequency, they will be exposed to the higher total waiting cost and if they try to decrease total waiting cost they do, they will be exposed to the higher total transportation cost. Thus, there is a trade-off between transportation cost and waiting cost and shippers need to find optimum point for minimizing their total cost. Shippers can have different due dates, responsibilities or priority for the goods to be delivered. So, shippers need to find out ways to manage their operations with a joint shipment scheme.

In this study, we study the case where there are multiple shippers making shipments from the same origin to the same destination. Shippers may collaborate and make joint shipments to the same destination for increasing utilization of the transportation assets and making more profit from shipments. We work on an environment where the shipment requests of the shippers arrive over time at a deterministic uniform constant rate, and the shippers consolidate their shipments on trucks that have sufficiently large capacity. The trucks are dispatched at a certain frequency to maximize the profit gained from the shipments. The joint shipments enables cost-saving from total transportation costs and these savings should be shared among all shippers. Each shipper potentially has different characteristics such as revenue, arrival rate of goods to be shipped or waiting cost. For a sustainable shipment consolidation initiation, shippers need to consent to their allocated profit and their primary examination will be comparing the allocated profit with profit they make with individual shipments. We focus on the joint shipment and profit allocation decisions.

To investigate the profit allocation of joint-shipment, we study cooperative and non-cooperative games of joint shipment. In the cooperative game, shippers act as a coalition and the total benefit is also considered beside the individual benefits of shippers. In non-cooperative game, each shipper acts as selfishly and make their joint shipment decisions according to only their benefits. We study both scenarios and then compare the profit allocation of the games through numerical analysis. That profit loss on the allocated profits is quantified by price of anarchy.

Our study aims to answer the following research questions under an uncapacitated and deterministic environment: (1) What are the necessary conditions for shippers having different characteristics to make profitable joint-shipments? (2) How do shippers operate optimally with joint-shipments? (3) Does there exist a desired scheme to allocate the profit? If exists, does it satisfy the properties of cooperative games and is it sustainable? (4) What is the setting for a non-cooperative game, does Nash Equilibria exist? (5) Does price of anarchy, the profit loss due to acting selfishly instead of coalition, exists?

The remaining chapters are organized as follows: In Chapter 2, we discuss the related studies from the literature. Then, we start our problem setting and present some structural results in Chapter 3. Here, 2 shipper and s shipper cases are analyzed. Next, In Chapter 4 and In Chapter 5, cooperative game and non-cooperative game are studied, respectively. Through the findings of these games, the price of anarchy is discussed in Chapter 6. Lastly, the remarks of our thesis and possible future works are shared in the conclusion part.

CHAPTER 2

LITERATURE REVIEW

After the realization of benefits of joint replenishment and/or shipment consolidation for organizations, the researches about these topics has increased and developed for different scenarios. Also, a major question emerged from these concepts is that how the benefits of collaboration will be shared. In the related literature, this problem has also worked in detail and there are several proposed solution approaches for different settings using cooperative or non-cooperative game theory methodologies. In the following sections, the studies that are related with our work are reviewed.

2.1 Cooperative Inventory Game

Meca et al. (2004) studies joint replenishment for multiple retailer demanding for same good from a single supplier. They study basic EOQ inventory model where the decision parameter is average inventory cost per time unit and the objective is to minimize the average cost per time unit of the inventory. They study optimal ordering policy for the coalition and the allocations of cost savings among the firms in coalition by cooperative game theory. They specified that the ordering cost as same and revealed information for all firms but the constant rates of demands and holding costs as private information. They found that sharing firm's average number of orders per time unit is enough for determining optimal joint ordering policy.

Körpeoğlu et al. (2012) studies a non-cooperative game for joint replenishment by multiple firms which are operating under an EOQ-like setting with inventory holding costs and demand rates. Each firm decides whether to replenish jointly and if so determines the contribution amount to ordering cost. There is an intermediary who determines the joint cycle time which is the lowest joint cycle time that can be financed with the joint contributions of the firms.It is found that when there is a single firm with lowest stand-alone cycle time, there is a unique undominated Nash equilibrium. That firm pays all ordering cost and others firms becomes free riders and enjoy free deliveries. The free riders's equilibrium cost is lower than their stand-alone costs. When there are multiple firms with the lowest stand-alone cycle time, there are multiple equilibria.In some of these equilibria, there can be also free rider firms but in any equilibrium consisting more than one contributor,all firms are strictly better than their individual replenishment case. In any case, the equilibrium joint cycle time is equal to the lowest stand-alone cycle time.

Güler, Körpeoğlu and Şen (2017) studies jointly replenishing multiple firms operating under EOQ like environment in a decentralized, non-cooperative setting where demand rate and inventory holding cost are private information. They seek to find a policy that determines the joint replenishment frequency and allocating the joint ordering costs through their reported stand-alone replenishment frequencies. Their first finding is that there does not exist a direct mechanism that can achieve efficiency, incentive compatibility, individual rationality and budget-balance. After that, they propose a two parameter mechanism which are used for determining the joint replenishment frequency and allocation of order costs based on firms' reports. They found that the order costs should be allocated uniformly for achieving efficiency. They clarify the conditions for constructive equilibrium, characterize and comparative statistics for the case the proposed two parameters are equal. They also show that the mechanism with smaller values of the two parameters which are equal to each other leads to more efficient outcomes and more defendable considering fairness.

In Timmer et al. (2013), companies is reviewing their inventory continuously and facing Poisson demand. Authors study to find cooperation strategies for the companies. They seek stable cost allocations of joint costs so that the companies become devoted to cooperation. Timmer et al. (2013) consider two cooperation policies for further comparisons. First one is that companies jointly ordering at joint inventory position reorder level. Second one is that ordering as soon as any of them reaches its reorder level. They provide explicit expressions of the joint costs under cooperation and individual allocated costs. Through their comparisons of the strategies and the non-cooperative case, they find that the individual constraints strategy has lowest cost for two or three companies. Based on their numerical experiences, stable cost allocations of joint cost exist proven by the distribution rule and the Shapley value. They also find that both cost allocation rules are robust to parameter changes and may provide benefits to companies with cooperation.

Dror and Hartman (2011) presented a general survey of cooperative inventory games. They examine deterministic and stochastic games. For deterministic games, they focus on Economic Order Quantity policies and for stochastic games, they are interested on Newsvendor-like centralization games. After they review the existing literature on these type games, they point out the potential future research questions not examined in the literature and they state that there would be a significant interest on documenting real life cases of cooperating competitors and providing empirical analysis for cost allocation rules of these competitors applied.

Satir et al.(2018) works on the problem of allocating vehicle capacity between two shipment order types which are dynamically consolidated. They model the problem as a continuous-time Markov Decision Process with the objective of minimizing total discounted cost.It is assumed that orders arrive according to compound Poisson process and there are two types of orders needed to be shipped which are expedited and regular. They propose quantity-based optimal threshold policies under particular conditions. In their models, they use two simplifications which are common in existing literature. Their state space takes only the accumulated orders in unit-order sizes and they use holding cost as a penalty to late deliveries of orders without hard due-date constraints.They implement the solution approach in two real life problems and they show that the approach outperformed the existing time policies.

2.2 Cooperative Transportation Game

Ergun et. al. (2007) study shipper collaboration by emphasizing on the logistics collaborative networks managed by a common computing platform to reduce "hidden costs" due to asset repositioning. They state that through collaboration shippers may form tours with higher utilization rates and less asset repositioning comparing to individual lanes. In their study, they focus on finding a set of tours connecting regularly executed truckload shipments to minimize asset repositioning. They formulate the lane covering problem which is a combinatorial optimization problem and propose several heuristic methods to solve and make a computational study on testing their solution approach.

Yılmaz and Savaseneril (2011) study collaboration among small shippers under uncertainty. Their focused problem is coalition formation among shippers with small-volume and stochastic shipments and designing fair allocation mechanisms to share vehicle capacity. They come up with a continuous time Markov Chain model with the objective of cost minimization. They compare the performance of the model with naive and myopic policies under different shipper characteristics and present the results for alternative scenarios. They claim that for a successful collaboration, the shipment volumes, the timings of shipments, tightness of delivery times and sensitivity to the late deliveries for each shipper should be analyzed.Also, they provide the conditions of the fair allocation of savings and stability of the coalition to prevent occurrence of sub-coalitions.

Pan et. al. (2019) propose a comprehensive survey of the development of horizontal collaborative transport (HCT) over the past ten years. They aim to identify research trends, gaps and propose some prospective lines of research questions for further studies.They developed a survey framework based on two axis: HCT solutions and implementation issues. They claim that their survey can be benefical for the industry stakeholders by providing guidelines for choosing a HCT solution and its possible implementation difficulties. They find that carrier alliance and flow controller collaboration were the most frequently studied but pooling and physical internet solutions are started to be attractive topics.

2.3 Cost Allocation in Coalition

Anily and Haviv (2007) work on a cost allocation rule under an infinite-horizon deterministic joint replenishment system where the setup transportation/reorder cost associated with a group of retailers placing an order at the same time equals some group-independent major setup cost plus retailer-dependent minor setup costs. They focus on the power-of-two policy and show that under the optimal power-of-two policy, the corresponding cooperative game satisfies concavity and the game is totally balanced game having nonempty core sets. They also prove that there are infinitely many core allocations.

Zhang (2008) study on the cost allocation problem for joint replenishment models. They consider a case where there is one warehouse multiple retailers and a sub-modular joint setup cost for replenishment. They focus on cost allocation of of the retailers under an power-of-two policy and they claim that the allocation problem can be represented as a cooperative game. They prove that the game has non-empty core sets. To prove the nonemptiness of the core sets, they use a duality approach.

2.4 Joint Replenishment

Jackson et. al. (1983) study on joint replenishment problem focusing on realistic and consistent reorder intervals in production or distribution systems. They develop a general dynamic programming formulation of the problem beside of the general statement assuming constant reorder intervals. They present an sorting algorithm based on the power-of-two restriction. They claim that the average annual cost of their proposed solution is within 6% of the general problem's long-run minimum average annual cost.

Yu-sheng and Zheng (1992) study on joint replenishment problem under several settings. There are constant but item specific demand rates and joint fixed procurement costs which are not separable. The replenishment intervals are the constant intervals which are the power-of-two multiples of some fixed or variable base planning period according to power-of-two policy. They develop algorithms to find optimal power-of-two policy. These algorithms also includes cost allocation to each items. They claim that with their proposed cost allocation scheme, the problem with separable costs and joint nonseperable costs are equivalent in the sense that both problem has same optimal solution set under power-of-two policy.

Bylka (2011) studies a setting consisting of one producer and multiple retailers. Demand is taken as constant and deterministic. Production rate is sufficient to satisfy the demand. They analyze special production and replenishment policies for vendor and buyers under a production distribution cycle. They assume that the product is instantly sent to buyers' stocks from the producer stock in discrete batches. The paper offers a non-cooperative game model with an objective of minimizing costs for all agents who need to choose sizes and numbers of production batches. For the cases where the vendor or the buyers control all the shipments, they found that there is Nash equilibrium in the sub-games. When there is a iterative coordination model starting with an agreement on the shipment number and after that deciding batch sizes non-cooperatively. In both games, agents try to choose strategies that minimizes their total costs. In these games, they proved that there are Nash and Stackelberg equilibrium.

He et al. (2017) study cost-allocation in a joint replenishment setting. They used the assumption that retailers order considering Power-of-Two policies where the replenishment interval of each retailer is an integer power-of-two times a base planning period. They study the problem as a non-cooperative game theory perspective which is considered as a complementary approach to the existing cooperative approaches. In their model, it is asked to all retailers to freely choose their own replenishment interval according to what is beneficial for themselves. Their proposed cost allocation rule is splitting the major setup cost equally to the retailers and the rule is preannounced to the retailers. So, they usually act on minimizing their cost with the anticipating the other retailers' response. They show that there is a Nash Equilibrium for their proposed cost allocation rule. They also compare the non-cooperative solution to centralized optimal solution and calculate efficiency loss according to social optimum solution. They conclude that their proposed cost-allocation rule can be an effective mechanism to set up a cost-efficient outcome that is also consistent with player's individual strategic behavior.

2.5 Price of Anarchy

Zhang et. al. (2018) study transportation network under two different routing scenarios. They compare the user-centric selfish routing policy versus the socially optimal and system-centric one. They consider an index to measure and increase efficiency:

the price of anarchy. It is defined as the ratio of the total travel latency cost under selfish routing over the corresponding quantity under socially optimal routing. So, they expect potential values of the price of anarchy greater or equal to one and the larger amount implies more inefficiency due to the selfish drivers. To estimate the price of anarchy they develop a model that derives the origins, destinations and user preferences of drivers from traffic data of a specific area. Using that, they claim several strategies that can decrease the price of anarchy and increase efficiency.

Perakis and Roels (2007) quantify the price of anarchy which is loss of efficiency from decentralizing operations that uses price-only contracts. They define the price of anarchy as the ratio of profits of centralized and decentralized supply chain. They study on the price of anarchy associated with different supply chain configurations such as push or pull inventory positioning, two or more stages, serial or assembly systems, single or multiple competing suppliers, and single or multiple competing retailers. After the analysis, they propose several findings. They find that in two stage supply chains, the price of anarchy is at least 1.71. Secondly, they claim that the efficiency of the supply chain generally decreases when the number of intermediaries increases. Thirdly, they claim that competition generally enhances the supply chain coordination. Lastly, they find that in pull configurations, the inventory risk is shared among the supply chain partners and it reduce the effects of double marginalization comparing to a push configuration.

Haughton, Rostami and Espahbod (2022) study on the price of anarchy in truckload transportation spot markets. Their approach is a combination mathematical optimization and behavioral experiments. They considers the price of anarchy with three metrics: total financial earnings, customer service level and eco-efficiency. They develop mathematical model and algorithms to make assignments of multi-day truck load. They also clarify how human behavior can raises the price of anarchy and present a platform for human behavior experiments to investigate the comparison of centralization versus decentralization. They claim that there is a price of anarchy on all three metrics: average of 16%–34% loss of earnings; 10% drop in on time delivery; and 18% drop in eco-efficiency. Also, they claim that price of anarchy for three metric worsens by human negotiations in spot market. They provide an illustration showing the doubled losses by human involvement.

2.6 Contribution of Our Study

In our study, we extend the researches by presenting an uncapacitated deterministic profit game by considering the cooperative game in the research of Meca et al. (2004) and the non-cooperative game in the research of Körpeoğlu et al. (2012). We consider the possibility of partially involvement of shippers to the joint shipment. We aim to quantify the price of anarchy by comparing the profits made in cooperative and non-cooperative games.

CHAPTER 3

PROBLEM SETTING AND STRUCTURAL RESULTS

We study shipment consolidation games where shippers can choose to operate fully or partially at each shipment cycle in an uncapacitated deterministic environment. We do structural analysis on the total profit of joint shipments for both 2 shipper and s shipper cases. Starting with the 2 shipper case, we analyze different scenarios changing on whether shippers can make shipments individually or not. We find out the parametric constraints that are needed to be satisfied to make profitable joint shipments. Next, we generalize our findings on s shippers, and express the total profit per unit time, optimal joint shipment frequency, and optimal operation levels for each shipper. After the analysis of total profit in detail, we study on allocation of the profit to shippers. We express a cooperative game setting where the shippers act as a coalition. An allocation scheme is presented for the shippers in the coalition and then the characteristic function is analyzed and the properties and the core of the cooperative game is shown. Then, we study a non-cooperative Nash game. The best response of the shippers and conditions under which Nash equilibrium exists are expressed. After the findings for both types of games, through numerical analysis, the price of anarchy is quantified by comparing the profit of each shipper under both games.

Suppose that there are n shippers that perform shipment activities between the same origin and destinations. To make use of the economies of scale and to make more profit, the items to be shipped consolidated. The items to be shipped arrive as shipment requests. The shipment requests by shippers assumed to arrive at a constant uniform rate. Arrival rate of requests for shipper i is λ_i. When "sufficient" amount of requests accumulate, a vehicle dispatches. Until dispatch, for shipper i a unit waiting cost of c_i is incurred per unit per time. For every dispatch(shipment), a transportation cost of A is incurred and shipper i earns r_i for each unit shipped.

In our problem setting, we analyze the benefits of joint shipment in a deterministic environment. It is assumed that vehicle capacity is sufficiently high.

We will first characterize the conditions, where the shippers are able to operate individually profitably. Let the time between the two consecutive shipments (dispatches) be T. Denote the total profit per unit time of shipper i, $TP_i(T)$.

$$TP_i(T) = r_i\lambda_i - \frac{A}{T} - \frac{c_i\lambda_i T}{2} \tag{3.1}$$

Shipper i is able to operate individually if $TP_i(T) > 0$. Lemma 3.0.1 presents the conditions for $TP_i(T) > 0$.

Lemma 3.0.1. *Let*$A^i = \frac{r_i^2\lambda_i}{2c_i}$ *be the threshold value for shipper* $i = 1, 2...n$. *When* $A < A^i$, $TP_i(T) > 0$, *e.g. shipper* i *is able to operate individually.*

Proof. To find the maximum individual profit with respect to T, the first order condition of TP_i is checked:

$$\frac{\partial TP_i(T)}{\partial T} = \frac{A}{T^2} - \frac{c_i\lambda_i}{2} = 0$$

Since the second order condition is negative and the function is concave, the optimal value of T is found as $T^* = \sqrt{\frac{2A}{c_i\lambda_i}}$. For $T = T^*$, $TP_i(T)$ is found as:

$$TP_i(T^*) = r_i\lambda_i - \sqrt{2Ac_i\lambda_i} \tag{3.2}$$

Therefore, shipment operating individually for shipper i only if $r_i\lambda_i > \sqrt{2Ac_i\lambda_i}$. We call them as "self-sufficient" shippers. So, to become a self-sufficient shipper, the following condition is need to be satisfied for shipper i:

$$A < \frac{r_i^2\lambda_i}{2c_i} = A^i \tag{3.3}$$

■

It is found that there is a critical level, defined as A^i, for shippers to become self-sufficient to make profitable shipments.

3.1 Two Shipper Setting

First, we analyze the case where $n = 2$ to have a complete understanding of the process.

Any interval of time that begins with shipment of orders and ends the instant before the next shipment is called a cycle for the setting. The cycles are regenerative and are stationary. There can be no profitable and preferable ways for all shippers to waiting until joint shipment due to the different characteristics. Revenues and waiting costs of per unit can be varied for all shippers and it directly affects shippers' profitability. In our setting, a shipper i may partially take place in a joint shipment cycle. It is a shipper's decision to maximize their profits from the time they allocated for operating with the joint shipment. We use p_i to denote the fraction of time that shipper i taken place in a cycle T.

For shipper 1 and shipper 2, given p_1 and p_2, individual cycle times T_1 and T_2 equal to Tp_1 and Tp_2, consecutively. In these cycle times, shippers are actively operating and their goods to be shipped are waiting until dispatch. Until the dispatch, shippers are keeping their goods in their stocks and waiting cost is incurred at that duration. When $p_i = 1$, shipper i is called as "full time shipper" and when $0 < p_i < 1$, shipper i is called as "partial time shipper". For the setting, shipper 1 is always a full time shipper.

For a partial time shipper, there is a time that they do not choose to operate with this group, Tp_1 - Tp_2. So, in that time, they can search and try to find a different shipment consolidation group or they can prefer to not operate in that times which is a more profitable scenario from their individual shipment case.

In Figure 3.1, cycles of shipper 1 and shipper 2 are shown as an example. Here, T_1 is greater than T_2 and shipper 2 operates with a lower fraction of time than shipper 1. The quantity waiting to ship increases with arrival rates of shippers, λ_1 and λ_2 per unit time. Despite of different level of arrival rates and quantities of goods to be shipped, they are able to make joint-shipments.

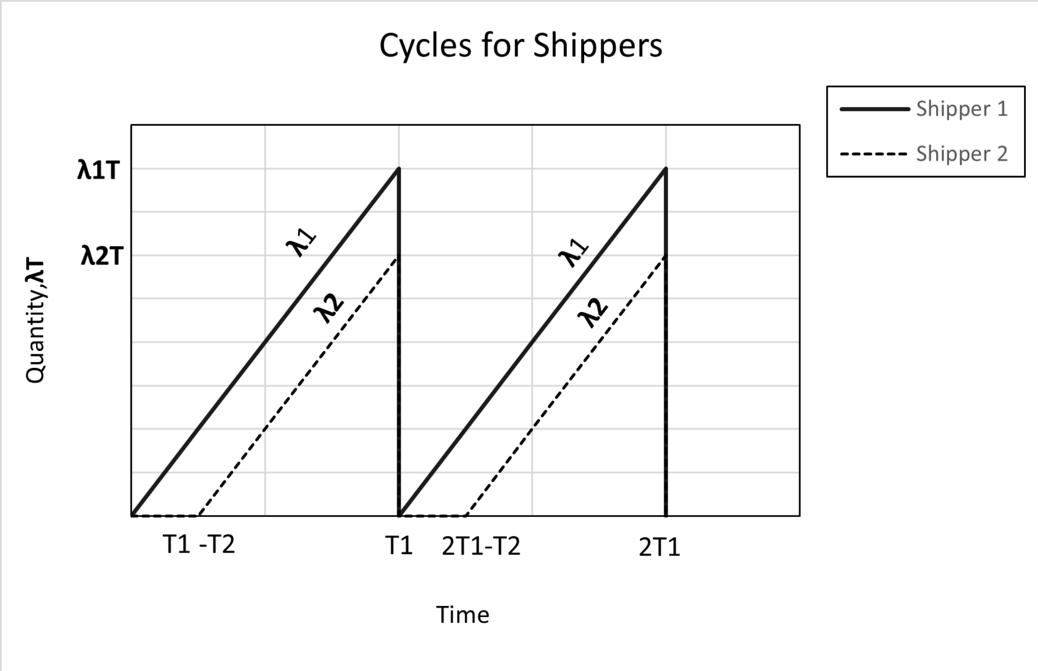

Figure 3.1: Cycles of each shipper when shippers make the dispatch decisions jointly, $T_1 > T_2$

Given T, one can express total profit per cycle as follows:

$$r_1\lambda_1 Tp_1 + r_2\lambda_2 Tp_2 - (A + \frac{c_1\lambda_1 T^2 p_1 p_1}{2} + \frac{c_2\lambda_2 T^2 p_2 p_2}{2}) \tag{3.4}$$

Dividing this by T, one gets $TP(T)$, which is total profit of shippers per unit time. It is the total cycle profit divided by the cycle length.

$$TP(T) = r_1\lambda_1 p_1 + r_2\lambda_2 p_2 - (\frac{A}{T} + \frac{c_1\lambda_1 Tp_1^2}{2} + \frac{c_2\lambda_2 Tp_2^2}{2}) \tag{3.5}$$

To maximize total profit that shippers get, the following optimization model is built. Here, T, p_1 and p_2 are decision variables and the model is non-linear.

(Profit) Max $TP(T, p_1, p_2)$= $r_1\lambda_1 p_1 + r_2\lambda_2 p_2 - (\frac{A}{T} + \frac{c_1\lambda_1 Tp_1^2}{2} + \frac{c_2\lambda_2 Tp_2^2}{2})$

s.t. $\quad 0 \leq p_1 \leq 1$

$0 \leq p_2 \leq 1$

$T > 0$

The objection function of the model is maximizing the profits of shipper 1 and shipper 2 under a joint-cycle time T and under individual operating times, Tp_1 and Tp_2.

The constraints of the model states the bounds on the variables. To determine the maximum total profit, $TP(T)$, one needs to determine optimal T, p_1 and p_2. Let T^* be the optimal T value for $TP(T)$ function and p_1^* and p_2^* be the optimal p_1 and p_2 values. We follow a two stage approach where we first determine $p_1^*$and p_2^* for a given T, then using these determine T^*.

Lemma 3.1.1. *For a given T, objective function of (Profit) model is concave in (p_1, p_2).*

Proof. Hessian matrix, $H(p_1, p_2)$ is set for verifying concavity. Hessian matrix of the objective function is a 2×2 matrix whose ij_th entry is $\frac{\delta^2 TP(T,p_1,p_2)}{\delta p_i \delta pj}$. When $TP(T, p_1, p_2)$ has continuous second-order partial derivatives for all variables and all non-zero principal minors have the same sign as $(-1)^k$ where k=1,2; then $TP(T, p_1, p_2)$ is concave. (Winston,2004)

The Hessian matrix is given below:

$$H(p_1, p_2) = \begin{bmatrix} -c_1\lambda_1 T & 0 \\ 0 & -c_2\lambda_2 T \end{bmatrix}$$

An ith principal minor of a matrix is the determinant of any matrix obtained by deleting $n - i$ rows and the corresponding $n - i$ columns of the matrix. (Winston,2004)

The first principal minors of $TP(T, p_1, p_2)$ are diagonal entries of the Hessian which are $-c_1\lambda_1 T$ and $-c_2\lambda_2 T$. These are both non positive.

The second principal minor $TP(T, p_1, p_2)$ is the determinant of H(p_1, p_2) and it is $c_1c_2\lambda_1\lambda_2 T^2$ and positive. Thus, TP is a concave function. ■

By proving concavity, it is determined that there is a unique maximum point of $TP(T)$. So, p_1^* and p_2^* exists. For a given T, the optimal values of p_1 and p_2 will be found through the KKT conditions. Let μ_1,μ_2, μ_3, μ_4 be the multipliers satisfying the Kuhn-Tucker(KKT) conditions. The related constraints are given in same order:$p_1 \leq 1$,$0 \leq p_1$,$p_2 \leq 1$,$0 \leq p_2$. Also, for the further analysis, w.l.o.g. we assume the following relation: $\frac{r_1}{c_1} > \frac{r_2}{c_2}$.

Lemma 3.1.2. *Optimal* $(\mu_1, \mu_2, \mu_3, \mu_4, p_1^*, p_2^*)$ *are characterized as follows:*

$$(\mu_1, \mu_2, \mu_3, \mu_4, p_1^*, p_2^*) = \begin{cases} (r_1\lambda_1 T - c_1\lambda_1, r_2\lambda_2 T - c_2\lambda_2, 0, 0, 1, 1) & T < \frac{r_2}{c_2} < \frac{r_1}{c_1} \\ (r_1\lambda_1 T - c_1\lambda_1, 0, 0, 0, 1, \frac{r_2}{c_2 T}) & \frac{r_2}{c_2} \leq T < \frac{r_1}{c_1} \\ (0, 0, 0, 0, \frac{r_1}{c_1 T}, \frac{r_2}{c_2 T}) & \frac{r_2}{c_2} < \frac{r_1}{c_1} \leq T \end{cases}$$

Proof. For a given T, the Kuhn-Tucker(KKT) conditions are checked for optimal solution. Since our problem is a maximization problem, if (p_1^*, p_2^*) is an optimal solution to the problem, then (p_1^*, p_2^*) must satisfy the following conditions and there must exist multipliers μ_1,μ_2, μ_3, μ_4 satisfying the conditions discussed below.

For a maximization problem $f(x_1, x_2, ..x_n)$ with n decision variables and m constraints in ($g_i(x_1, x_2, ..x_n) \leq b_i \quad i = 1, 2, ...m$), if $\bar{x} = (\bar{x_1}, \bar{x_2}, .., \bar{x_n})$ is an optimal solution, then $\bar{x}$ must satisfy the constraints of the model, and there must exist $\bar{\lambda_1}, \bar{\lambda_2}...., \bar{\lambda_n}$ satisfying: (Winston, 2004)

$$\frac{\delta f(\bar{x})}{x_j} - \sum_{i=1}^{i=m} \bar{\lambda_i} \frac{\delta g_i(\bar{x})}{\delta x_j} = 0 \qquad j = 1, 2, ...n$$
$$\bar{\lambda_i}[b_i - g_i(\bar{x_1})] = 0 \qquad i = 1, 2, ...m$$
$$\bar{\lambda_i} \geq 0 \qquad i = 1, 2, ...m$$

Constraints which emerge from the KKT conditions are given below:

$$p_1^* = \frac{r_1\lambda_1 - \mu_1 + \mu_3}{c_1\lambda_1 T} \tag{3.6}$$

$$p_2^* = \frac{r_2\lambda_2 - \mu_2 + \mu_4}{c_2\lambda_2 T} \tag{3.7}$$

$$\mu_1 - \mu_1 p_1 = 0 \tag{3.8}$$

$$\mu_2 - \mu_2 p_2 = 0 \tag{3.9}$$

$$\mu_3 p_1 = 0 \tag{3.10}$$

$$\mu_4 p_2 = 0 \tag{3.11}$$

$$\mu_i \geq 0 \forall i \tag{3.12}$$

For a given $0 < T < \infty$, as a result of these conditions, the following cases are obtained. Let w.l.o.g. $\frac{r_1}{c_1} > \frac{r_2}{c_2}$. For μ_i where $i = 1, 2, 3, 4$, there are 16 cases to analyze for corresponding value positions which are 0 or ≥ 0.

1. When $\mu_1 = 0$, $\mu_2 = 0$, $\mu_3 = 0$, $\mu_4 = 0$; from Equations 3.6 and 3.7, $p_i^* = \frac{r_i}{c_i T}$ for i=1,2, where $\frac{r_2}{c_2} < \frac{r_1}{c_1} \leq T$.

2. When $\mu_1 > 0$, $\mu_2 = 0$, $\mu_3 = 0$, $\mu_4 = 0$; from Equations 3.8 and 3.7, $p_1 = 1$ and $p_2^* = \frac{r_2}{c_2 T}$ where $\frac{r_1}{c_1 T} > 1$ and $\frac{r_2}{c_2} \leq T < \frac{r_1}{c_1}$.

3. When $\mu_1 > 0$, $\mu_2 > 0$, $\mu_3 = 0$, $\mu_4 = 0$; from Equations 3.8 and 3.9, $p_1^* = 1$ and $p_2^* = 1$ where $T < \frac{r_2}{c_2} < \frac{r_1}{c_1}$.

   Following cases have no real or considerable solution. The cases and reasons about why they are not considered are given:

4. When $\mu_1 = 0$, $\mu_2 > 0$, $\mu_3 >= 0$ and $\mu_4 > 0$, since $\frac{r_1}{c_1} > \frac{r_2}{c_2}$, from Equation 3.7, this case can not happen.

5. Note that, from Equations 3.8 and 3.9, there will be no solution where $\mu_1 > 0$ and $\mu_3 > 0$ or $\mu_2 > 0$ and $\mu_4 > 0$ since they can not exist for any corresponding p_1 and p_2 values.

6. Note that, from Equations 3.6 and 3.7, there will be no solution where $\mu_1 = 0$ and $\mu_3 > 0$ or $\mu_2 = 0$ and $\mu_4 > 0$ since p_i^* can not be equal to 0 when r_i,c_i,T and μ_3 or μ_4 are all positive.

$\blacksquare$

Next, we determine T^*.

Corollary 3.1.1. *$TP(T)$ is a piece wise function defined as where TP_a is equal to $(r_1\lambda_1 + r_2\lambda_2) - (\frac{A}{T} + \frac{(c_1\lambda_1 + c_2\lambda_2)T}{2})$, TP_b is equal to $(\frac{r_2^2\lambda_2}{2c_2} - A)(\frac{1}{T}) + r_1\lambda_1 - (\frac{c_1\lambda_1 T}{2})$ and TP_c is equal to $(\frac{r_1^2\lambda_1}{2c_1} + \frac{r_2^2\lambda_2}{2c_2} - A)(\frac{1}{T})$*

$$TP(T) = \begin{cases} TP_a & T < \frac{r_2}{c_2} < \frac{r_1}{c_1} \\ TP_b & \frac{r_2}{c_2} \le T < \frac{r_1}{c_1} \\ TP_c & \frac{r_2}{c_2} < \frac{r_1}{c_1} \le T \end{cases}$$

Proof. As the optimal p_1 and p_2 values are given for the specified regions in Lemma 3.1.2, TP_a, TP_b and TP_c are found by solving $p = p^*$ in the specified region. ■

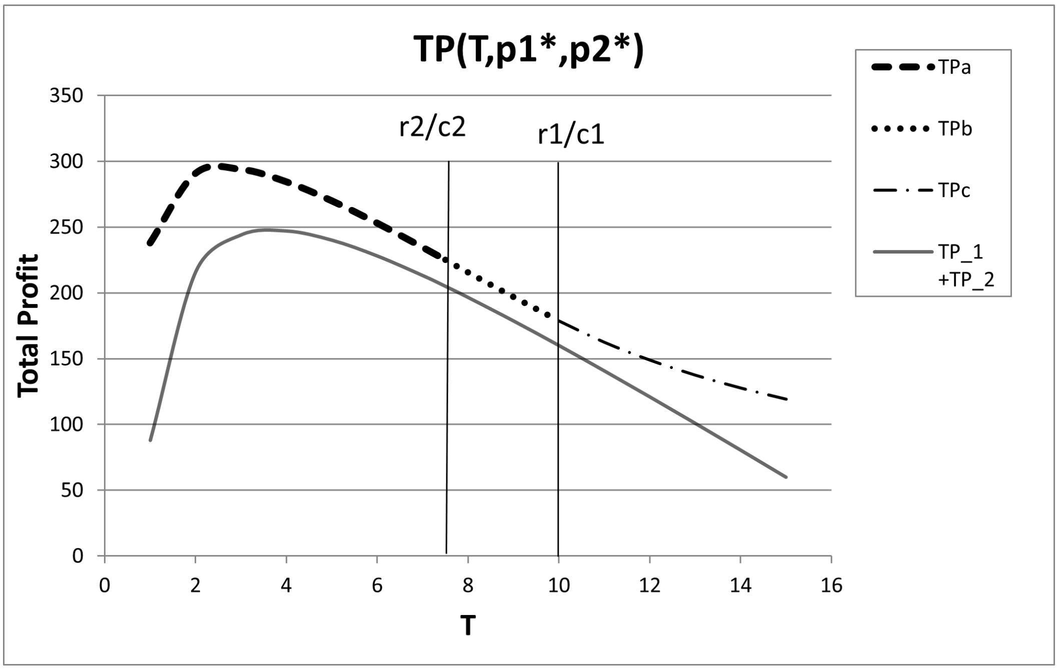

Figure 3.2: Behavior of TP(T) with respect to T, and comparison with individual shipments

Figure 3.2 shows an example case for comparison of total profit with respect to different cycle times where $A = 150$, $(r_1, c_1, \lambda_1) = (40, 4, 8)$ and $(r_2, c_2, \lambda_2) = (30, 4, 3)$. Here, it is shown that $\frac{r_2}{c_2}$ and $\frac{r_1}{c_1}$ are critical levels for TP regions. TP_a, TP_b and TP_c is shown at only in functions true regions. Also, shippers's total joint-shipment profits are higher than their total individual shipment profits for any cycle time. Observe that the slopes of the function at connection points are same and there is a global maximum total profit value.

Proposition 3.1.1. *$TP(T, p_1^*, p_2^*)$ is continuous and differentiable for all T.*

Proof. Values of $\lim_{T \to \frac{r_2}{c_2}} TP_a(T)$ and $TP_b(T = \frac{r_2}{c_2})$ are equal to each other since;

$$\lim_{T \to \frac{r_2}{c_2}} TP_a(T) = TP_b(T = \frac{r_2}{c_2}) = r_1\lambda_1 + r_2\lambda_2 - (\frac{(c_1\lambda_1 + c_2\lambda_2)r_2}{2c_2} + \frac{Ac_2}{r_2}) \quad (3.13)$$

When the derivative of the TP from both directions at $T = \frac{r_2}{c_2}$ is checked, it is observed that they are equal to each other:

$$\lim_{T \to \frac{r_2}{c_2}^+} \frac{\partial TP_b(T))}{\partial T} = \frac{Ac_2^2}{r_2^2} - \frac{c_1\lambda_1 + c_2\lambda_2}{2} \quad (3.14)$$

$$\lim_{T \to \frac{r_2}{c_2}^-} \frac{\partial TP_a(T))}{\partial T} = \frac{Ac_2^2}{r_2^2} - \frac{c_1\lambda_1 + c_2\lambda_2}{2} \quad (3.15)$$

Values of $\lim_{T \to \frac{r_1}{c_1}} TP_b(T)$ and $TP_c(T = \frac{r_1}{c_1})$ are equal to each other since;

$$\lim_{T \to \frac{r_1}{c_1}} TP_b(T) = TP_c(T = \frac{r_1}{c_1}) = (\frac{r_2^2\lambda_2}{2c_2} - A)(\frac{c_1}{r_1}) - \frac{r_1\lambda_1}{2} \quad (3.16)$$

Directions of TP_b and TP_c while converging to $T = \frac{r_1}{c_1}$ are equal to each other since;

$$\lim_{T \to \frac{r_1}{c_1}^-} \frac{\partial TP_c(T))}{\partial T} = \lim_{T \to \frac{r_1}{c_1}^+} \frac{\partial TP_b(T = \frac{r_1}{c_1}^-))}{\partial T} = \frac{c_1^2}{r_1^2}(A - \frac{r_1^2\lambda_1}{2c_1} - \frac{r_2^2\lambda_2}{2c_2}) \quad (3.17)$$

■

Thus, it is now verified that there is $TP(T, p_i^*, p_j^*)$ is continuous and differentiable in T. So, for finding optimal cycle time, the regions and their boundaries can be analyzed.

Proposition 3.1.2. *When* $(\frac{r_1^2\lambda_1}{2c_1} + \frac{r_2^2\lambda_2}{2c_2} - A) \geq 0$, *$TP_c(T)$ is decreasing in T, so the T value that maximizes $TP_c(T)$ is $\frac{r_1}{c_1}$ and shipper 1 is full time shipper where* $\frac{r_2}{c_2} < \frac{r_1}{c_1} \leq T$.

Proof. When $(\frac{r_1^2\lambda_1}{2c_1} + \frac{r_2^2\lambda_2}{2c_2} - A) > 0$, value of $TP_c(T)$ is decreasing in T for $\frac{r_2}{c_2} < \frac{r_1}{c_1} \leq T$. It can be checked from the derivative:

$$\frac{\partial TP_c(t)}{\partial T} = \frac{(\frac{r_1^2\lambda_1}{2c_1} + \frac{r_2^2\lambda_2}{2c_2} - A)}{T^2} < 0 \quad (3.18)$$

When $\frac{r_2}{c_2} < \frac{r_1}{c_1} \leq T$, $TP = TP_c(T)$ and $p_1^* = \frac{r_1}{c_1 T^*}$. Plugging T^* into p_1^* formula gives $p_1^* = 1$ value for $TP_c(T)$ function. ■

Note that when $(\frac{r_1^2\lambda_1}{2c_1} + \frac{r_2^2\lambda_2}{2c_2} - A) < 0$, $\frac{\partial TP_c(T)}{\partial T}$ is positive and there will be no optimal extreme point in $TP_c(T)$ as T^* converges to ∞, meaning that there will be no shipments. So, for all joint-shipments, optimal cycle time can be searched in $TP_a(T)$ and $TP_b(T)$ the region of $T < \frac{r_1}{c_1}$.

Let T_S^F be the T value that maximizes$TP_a(T)$ and T_L^F be the T value that maximizes$TP_b(T)$. Let A^2 be $\frac{r_2^2\lambda_2}{2c_2}$.

Proposition 3.1.3. $T^* = \begin{cases} T_S^F = \sqrt{\frac{2A}{c_1\lambda_1+c_2\lambda_2}} & ifT_S^F \leq \frac{r_2}{c_2} \\ T_L^F = \sqrt{(\frac{2}{c_1\lambda_1})(A - A^2)} & ifT_L^F > \frac{r_2}{c_2} and A \geq A^2 \end{cases}$

When $A < A^2$, T_L^F is not defined. Thus, $T^* = T_S^F \leq \frac{r_2}{c_2}$.

Proof. T_S^F and T_L^F values are found by the FOC of the $TP_a(T)$ and $TP_b(T)$ functions, respectively.

$$\frac{\partial TP_a(T)}{\partial T} = \frac{A}{T^2} - \frac{c_1\lambda_1 + c_2\lambda_2}{2} = 0 \tag{3.19}$$

Note that TP_a is concave in T. Thus from FOC:

$$T_S^F = \sqrt{\frac{2A}{c_1\lambda_1 + c_2\lambda_2}} \tag{3.20}$$

When $T_S^F \leq \frac{r_2}{c_2}$, where TP_a is defined, $T^* = T_S^F$ since TP is continuous and differentiable(see Proposition 3.1.1). Taking derivative of $TP_b(T)$ with respect to T:

$$\frac{\partial TP_b(T)}{\partial T} = (\frac{-r_2^2\lambda_2}{c_2} + A)(\frac{1}{T^2}) - \frac{c_1}{\lambda_1} = 0 \tag{3.21}$$

$$T_L^F = \sqrt{(\frac{2}{c_1\lambda_1})(A - \frac{2r_2^2c_2}{\lambda_2})} = \sqrt{(\frac{2}{c_1\lambda_1})(A - A^2)} \tag{3.22}$$

When T_L^F exists and $T_L^F > \frac{r_2}{c_2}$, where TP_b is defined, $T = T_L^F$ since TP is continuous and differentiable(see Proposition 3.1.1); ■

So, it is shown that T^* is set according to T_S^F and T_L^F values and their relation with $\frac{r_2}{c_2}$. It also indicates that whether shipper 2 is full time shipper or partial time shippers.

Corollary 3.1.2. *Let A^3 be the value of A that makes the derivative of TP(T) at $T = \frac{r_2}{c_2}$ equal to zero. Then, $A^3 = \frac{r_2^2}{2c_2^2}(c_1\lambda_1) + A^2$ and $A^3 > A^2$.*

Proof. When we equate the first derivative of TP at $T = \frac{r_2}{c_2}$ to 0, we obtain the following:

$$\frac{\partial TP(T)}{\partial T} = \frac{Ac_2^2}{r_2^2} - \frac{c_1\lambda_1 + c_2\lambda_2}{2} = 0. \tag{3.23}$$

$$A^3 = \frac{r_2^2}{2c_2^2}(c_1\lambda_1) + \frac{r_2^2\lambda_2}{2c_2} \tag{3.24}$$

$$A^3 = \frac{r_2^2}{2c_2^2}(c_1\lambda_1) + A^2 \tag{3.25}$$

■

A^3 is the significant level for A value. It is the maximum level that shipper 2 is optimally full time shipper. If A is greater than A^3 value, then T^* is greater than $\frac{r_2}{c_2}$.

Proposition 3.1.4. *T^*, $p_1^*$and p_2^* values for corresponding A values are given below:*

- $T^* = \begin{cases} T_S^F & A < A^3 \\ T_L^F & A \geq A^3 \end{cases}$
- $(p_1^*, p_2^*) = \begin{cases} (1, 1) & A < A^3 \\ (1, \frac{r_2}{T_L^F c_2}) & A \geq A^3 \end{cases}$

The optimal profit function is given by:

$$TP^* = \begin{cases} r_1\lambda_1 + r_2\lambda_2 - \sqrt{2A(c_1\lambda_1 + c_2\lambda_2)} & A < A^3 \\ r_1\lambda_1 - \sqrt{2c_1\lambda_1(A - A^2)} & A \geq A^3 \end{cases}$$

Proof. In Corollary 3.1.2,it is shown that when $A = A^3$, $T^* = \frac{r_2}{c_2}$. For $A < A^3$ leads $T^* = T_S^F$ and for $A > A^3$ leads $T^* = T_L^F$.

$p_1^*$and p_2^* is set by Equation 3.1.2 and relation of A^3 and $\frac{r_2}{c_2}$.

After determining T^*, $p_1^*$and p_2^*, TP^* is found by plugging in the variables.

■

It is shown that for a 2 shipper setting; T^*, p_1^* and p_2^* can be determined with the comparison of A and A^3. This also enables to find out TP^* value.

3.2 Profitability Analysis for Two Shipper Setting

After that, we start to analyze different cases where shippers's self-sufficiency differs. That will enable to clarify the conditions for making profitable joint-shipments. The following table provides value and explanations of the parameters, which is going to be discussed in following parts.

Table 3.1: Table of Parameters, Threshold values and their explanations

Parameter	**Value**	**Explanation**
A^1	$\frac{r_1^2\lambda_1}{2c_1}$	A threshold value of A for the first shipper's profitable individual shipment.(self-sufficiency)
A^2	$\frac{r_2^2\lambda_2}{2c_2}$	A threshold value of A for the second shipper's profitable individual shipment.
A^3	$\frac{r_2^2}{2c_2^2}(c_1\lambda_1) + A^2$	A corresponding value for A makes the derivative of the total profit function at $\frac{r_2}{c_2}$ equal to 0.
A^4	$A^2 + \frac{r_1 r_2 \lambda_2}{2c_1}$	A threshold value of A for profitability of joint shipment in several cases.
A^5	$\frac{A^1}{2} + A^2$	A threshold value of A for profitability of joint shipment in several cases.
A^6	$\frac{r_2^2\lambda_2}{2[2c_1\lambda_1+c_2\lambda_2-2\sqrt{c_1\lambda_1(c_1\lambda_1+c_2\lambda_2)}]}$	A threshold value of A for profitability of joint shipment in several cases.
TD	$\sum_i(r_i\lambda_i - \sqrt{2Ac_i\lambda_i})$	Optimal total profit when shippers operate individually .

3.2.1 Self-sufficient shippers

Self-sufficient shipper means that the shipper can make shipments profitably and individually.(See Equation 3.3) In that case, we study joint-shipment for two self-sufficient shippers.

Proposition 3.2.1. *When $A < A^2$; $TP(T^*) > TD$, $(p_1^*, p_2^*) = (1,1)$ and $T^* = T_S^F$.*

Proof. When $A < A^2$, both shippers are self-sufficient.

- From Proposition 3.1.4,$T^* = T_S^F$ and $(p_1^*, p_2^*) = (1,1)$ since $A < A^2 < A^3$.
- To prove profitability, $TP(T^*) > TD$ is checked.
  From Proposition 3.1.4, it is found that $TP(T^*) = r_1\lambda_1 + r_2\lambda_2 - \sqrt{2A(c_1\lambda_1 + c_2\lambda_2)}$.
  Also, optimal individual profit function is $TP_i = r_i\lambda_i - \sqrt{2Ac_i\lambda_i}$ for shipper i=1,2.

$$r_1\lambda_1 + r_2\lambda_2 - \sqrt{2A(c_1\lambda_1 + c_2\lambda_2)} > r_1\lambda_1 - \sqrt{2Ac_1\lambda_1} + r_2\lambda_2 - \sqrt{2Ac_2\lambda_2} \tag{3.26}$$

$$\sqrt{2A(c_1\lambda_1 + c_2\lambda_2)} < \sqrt{2Ac_1\lambda_1} + \sqrt{2Ac_2\lambda_2} \tag{3.27}$$

$$2A(c_1\lambda_1 + c_2\lambda_2) < 2Ac_1\lambda_1 + 2Ac_2\lambda_2 + 2A\sqrt{c_1\lambda_1 c_2\lambda_2} \tag{3.28}$$

$$0 < 2A\sqrt{c_1\lambda_1 c_2\lambda_2} \tag{3.29}$$

  So, it is verified that joint-shipment is more profitable than the profit obtained when shippers operate individually when $A < A^2$.

∎

3.2.2 Not self-sufficient shippers

Here, we study a joint-shipment case where both shippers are not self-sufficient.

Proposition 3.2.2. *When $A > A^1$, $A^3 > A > A^2$ and $A < A^4$; $TP(T^*) > TD$, $(p_1^*, p_2^*) = (1,1)$ and $T^* = T_s^*$.*
When $A > A^4$, joint-shipment is not profitable and the shippers will not be able to do joint-shipment.

Proof. • When $A < A^2$, from Proposition 3.1.4, it is found that $T^* = T_S^F$ and $(p_1^*, p_2^*) = (1, 1)$ since $A > A^1$, $A^3 > A > A^2$.

- To prove profitability, $TP(T^*) > TD$ is checked.
  From Proposition 3.1.4, $TP(T^*) = r_1\lambda_1 + r_2\lambda_2 - \sqrt{2A(c_1\lambda_1 + c_2\lambda_2)}$.$TD$ equals to 0, since for $A > A^2$, shippers are not able to do profitable shipments. So, we checked the following inequality:

$$r_1\lambda_1 + r_2\lambda_2 - \sqrt{2A(c_1\lambda_1 + c_2\lambda_2)} > 0 \tag{3.30}$$

$$(r_1\lambda_1)^2 + (r_2\lambda_2)^2 + 2r_1\lambda_1 r_2\lambda_2 > 2A(c_1\lambda_1 + c_2\lambda_2) \tag{3.31}$$

$$\frac{(r_1\lambda_1)^2 + (r_2\lambda_2)^2 + 2r_1\lambda_1 r_2\lambda_2}{\lambda_1\lambda_2 c_1 c_2} > \frac{2A(c_1\lambda_1 + c_2\lambda_2)}{\lambda_1\lambda_2 c_1 c_2} \tag{3.32}$$

$$\frac{A^1}{\lambda_2 c_2} + \frac{A^2}{\lambda_1 c_1} + \frac{r_1 r_2}{c_1 c_2} > \frac{A}{\lambda_1 c_1} + \frac{A}{\lambda_2 c_2} \tag{3.33}$$

$$\frac{r_1 r_2}{c_1 c_2} > \frac{A - A^2}{\lambda_1 c_1} + \frac{A - A^1}{\lambda_2 c_2} \tag{3.34}$$

It is known that $A^3 > A > A^2$ and $A^3 > A > A^1$ due to not self-sufficiency conditions and the condition for A value. Also, $\frac{r_1}{c_1} > \frac{r_2}{c_2}$ is the assumption of the setting.

$$\frac{A^3 - A^2}{c_1\lambda_1} = \frac{r_2^2}{2c_2} < \frac{r_1 r_2}{2c_1 c_2} \tag{3.35}$$

Since, $A < A^3$ in this case, $\frac{A - A^2}{c_1\lambda_1}$ is also less than $\frac{r_1 r_2}{2c_1 c_2}$.

So, we found the profitability condition which is $A < A^4$by following arrangements;

$$\frac{r_1 r_2}{2c_1 c_2} + \frac{r_1 r_2}{2c_1 c_2} > \frac{A - A^2}{c_1\lambda_1} + \frac{A - A_1}{\lambda_2 c_2} \tag{3.36}$$

$$\frac{r_1 r_2}{2c_1 c_2} > \frac{A - A^2}{c_1\lambda_1} \tag{3.37}$$

$$A < \frac{r_1 r_2 \lambda_2}{2c_1} + A^1 = A^4 \tag{3.38}$$

So, when $A < A^4$, joint-shipment is profitable.

■

When shippers are not self-sufficient, there can possibly be some settings where joint-shipments are also not profitable.

Proposition 3.2.3. *When $A > A^1$, $A^5 > A > A^3$; $TP(T^*) > TD$, $(p_1^*, p_2^*) = (1, \frac{r_2}{T_L^* c_2})$ and $T^* = T_L^*$.*
When $A > A^5$, joint-shipment is unprofitable and the shippers will not be able to do joint-shipment.

Proof. When $A > A^1$, $A^5 > A > A^3$;

- From Proposition 3.1.4, it can be clarified that $T^* = T_L^*$ and $(p_1^*, p_2^*) = (1, \frac{r_2}{T_L^* c_2})$.
- To prove profitability, $TP(T^*) > TD$ is checked.
  From Proposition 3.1.4, it is found that $TP(T^*) = r_1\lambda_1 - \sqrt{2c_1\lambda_1(A - A^2)}$. Also, sum of individual profits equal to 0, since they are not be able to do profitable shipments. So, we checked following inequality:

$$r_1\lambda_1 - \sqrt{2c_1\lambda_1(A - A^2)} > 0 \tag{3.39}$$

Since we have $A > A^3$ and $A^3 - A^2 = \frac{r_2^2}{2c_2^2}(c_1\lambda_1)$, it can be said that $A - A^2 = \frac{r_2^2}{2c_2^2}(c_1\lambda_1) + \epsilon$, where $\epsilon > 0$.

$$r_1\lambda_1 - \sqrt{2c_1\lambda_1(\frac{r_2^2}{2c_2^2}(c_1\lambda_1) + \epsilon)} > 0 \tag{3.40}$$

$$(r_1\lambda_1)^2 > \frac{r_2^2 c_1^2 \lambda_1^2}{2c_2^2} + \frac{c_1\lambda_1 c_2^2 \epsilon}{c_2^2} \tag{3.41}$$

$$(r_1\lambda_1)^2 > \frac{r_2^2 c_1^2 \lambda_1^2 + c_1\lambda_1 c_2^2 \epsilon}{2c_2^2} \tag{3.42}$$

$$\frac{r_1^2\lambda_1^2 c_2^2 - r_2^2 c_1^2 \lambda_1^2}{c_1\lambda_1 c_2^2} > \epsilon \tag{3.43}$$

So, when ϵ is less than $\frac{r_1^2\lambda_1^2 c_2^2 - r_2^2 c_1^2 \lambda_1^2}{c_1\lambda_1 c_2^2}$, joint shipment is profitable. Since $A^5 = \frac{A^1}{2} + A^2 = A^3 + \frac{r_1^2\lambda_1}{c_1} - \frac{r_2^2\lambda_1 c_1}{c_2^2}$, it is proven that joint shipment is profitable under $A^5 > A > A^3$ condition.

∎

3.2.3 One self-sufficient shipper and One not self-sufficient shipper

The last case we study in this section is one self-sufficient shipper and one not self-sufficient shipper. In this case, the self-sufficient shipper is always taken as the first shipper. The conditions of each result is discussed in following theorems.

Proposition 3.2.4. *When $A < A^1$, $A > A^2$, $A < A^3$ and $A < A^6$; $TP(T^*) > TD$, $(p_1^*, p_2^*) = (1, 1)$ and $T^* = T_S^*$.*

Proof. When $A > A^2$, $A < A^3$ and $A < A^6$;

- From Proposition 3.1.4, it can be clarified that $T^* = T_S^*$ and $(p_1^*, p_2^*) = (1, 1)$.
- To prove profitability, $TP(T^*) > TD$ is checked.
  From Proposition 3.1.4, it is found that $TP(T^*) = r_1\lambda_1 + r_2\lambda_2 - \sqrt{2A(c_1\lambda_1 + c_2\lambda_2)}$. Also, individual profit functions are $TD = TP_1 = r_1\lambda_1 - \sqrt{2Ac_1\lambda_1}$ since shipper 2 is not able to do shipment.So, we checked following inequality:

$$r_1\lambda_1 + r_2\lambda_2 - \sqrt{2A(c_1\lambda_1 + c_2\lambda_2)} > r_1\lambda_1 - \sqrt{2Ac_1\lambda_1} \tag{3.44}$$

$$r_2\lambda_2 > \sqrt{2A(c_1\lambda_1 + c_2\lambda_2)} - \sqrt{2Ac_1\lambda_1} \tag{3.45}$$

$$\frac{r_2\lambda_2}{\sqrt{(c_1\lambda_1 + c_2\lambda_2)} - \sqrt{c_1\lambda_1}} > \sqrt{2A} \tag{3.46}$$

$$\frac{r_2^2\lambda_2^2}{2[2c_1\lambda_1 + c_2\lambda_2 - 2\sqrt{c_1\lambda_1(c_1\lambda_1 + c_2\lambda_2)}]} = A^6 > A \tag{3.47}$$

  So, we reach out the condition defined in theorem. When $A < A^1$, $A > A^2$, $A < A^3$ and $A < A^6$, the total profit of joint shipment is bigger than individual profits and both of them are full time shippers.

■

The following theorem considers joint-shipment of a full time shipper and a partial time shipper.

Theorem 3.2.1. *When $A < A^1$, $A > A^2$ and $A > A^3$; $TP(T^*) > TD$, $(p_1^*, p_2^*) = (1, \frac{r_2}{T_L^* c_2})$ and $T^* = T_L^*$.*

Proof. When $A < A^1$, $A > A^2$ and $A > A^3$;

- From Proposition 3.1.4, it can be clarified that $T^* = T_L^*$ and $(p_1^*, p_2^*) = (1, \frac{r_2}{T_L^* c_2})$.
- To prove profitability, $TP(T^*) > TD$ is checked.
  From Proposition 3.1.4, it is found that $TP(T^*) = r_1\lambda_1 - \sqrt{2c_1\lambda_1(A - A^2)}$. Also, individual profits are $TD = TP_1 = r_1\lambda_1 - \sqrt{2Ac_1\lambda_1}$ since shipper 2 is not able to do shipment.So, we checked following inequality:

$$r_1\lambda_1 - \sqrt{2c_1\lambda_1(A - A^2)} > r_1\lambda_1 - \sqrt{2Ac_1\lambda_1} \tag{3.48}$$

$$\sqrt{2c_1\lambda_1(A - A^2)} < \sqrt{2Ac_1\lambda_1} \tag{3.49}$$

$$A - A^2 < A \tag{3.50}$$

  So, the inequality defined by 3.48 is proved. When $A < A^1$, $A > A^2$ and $A > A^3$, second shipper is partial time shipper and total profit of joint-shipment is higher than the sum of individual shipments.

■

So, we anaylze possible cases where shippers can take

3.3 s Shipper Setting

Next, we analyze the case where there are s shippers instead of two to derive a more general solution approach. Any interval of time that begins with shipment of orders and ends at the instant before the next shipment is called a cycle for this setting also. The cycles are regenerative and are stationary.

Let p_i be the fraction of time for which shipper i operates and a given cycle time T for shipper i. Then, their individual cycle times T_i equals to Tp_i. In that cycle time, shippers are actively operating and their goods to be shipped are waiting until dispatch. Until the dispatch, shippers are keeping their goods in their stocks and waiting cost is incurred at that duration. There can be no profitable and preferable ways for all shippers to waiting until joint shipment due to different characteristics.

Revenues and waiting costs of per unit can be varied for all shippers and it directly affects shippers' profitability. When $p_i = 1$,shipper i is called as "full time shipper" and when $0 < p_i < 1$, shipper i is called as "partial time shipper". There is a relation among shippers such that $\frac{r_i}{c_i} > \frac{r_j}{c_j}$ where the shipper, that has biggest $\frac{r}{c}$ is assigned index of 1.

Given T, one can express total profit per unit time, $TP(T)$, as follows:

$$TP(T) = \sum_{i=1}^{s} r_i \lambda_i p_i - (\frac{A}{T} + \sum_{i=1}^{s} \frac{c_i \lambda_i T p_i^2}{2}) \tag{3.51}$$

So, the problem can be build as the following non linear programming model:

(Profit-s) Max z = $\sum_{i=1}^{s} r_i \lambda_i p_i + -(\frac{A}{T} + \sum_{i=1}^{s} \frac{c_i \lambda_i T p_i^2}{2})$

s.t. $0 \leq p_i \leq 1 \; \forall i \in 1, 2, 3..s$

$T \geq 0$

To determine the maximum total profit, TP, one needs to determine optimal T and p_i variables.The same two stage approach where we used in two player setting is applied: First determine p_i values for a given$T$, then using these determine T^*.

3.3.1 Finding optimal p_i's for a given T

To determine that the TP function for a given T is concave, Hessian matrix, $H(p_i, p_j)$ is set where $i \in 1, 2, 3..s$. $H(p_i, p_j)$ is a $s \times s$ matrix whose ijth entry is $\frac{\delta^2 TP(T)}{\delta p_i \delta pj}$. The Hessian matrix is given below:

$$H(p_i, p_j) = \begin{bmatrix} -c_1\lambda_1 T & . & . & . & 0 \\ 0 & -c_2\lambda_2 T & . & . & 0 \\ 0 & 0 & -c_i\lambda_i T & . & 0 \\ 0 & 0 & . & . & 0 - c_i\lambda_i T \end{bmatrix}$$

When $TP(T)$ has continuous second-order partial derivatives for all variables and all non-zero principal minors have the same sign as $(-1)^k$ where$k = 1, 2..s$; then $TP(T)$

is concave. (Winston,2004). Here, the first principal minors are diagonal entries of the $H(p_i, p_j)$, $-c_i\lambda_i T$. These are both non positive. The second principal minor is the determinant of$H(p_i, p_j)$ and it is $c_i c_j \lambda_i \lambda_j T^2 > 0$ .To generalize, if k is odd number, the kth principal minor is non positive else if k is even number, it is non negative. Thus, TP is a concave function.

For a given T, the optimal values of p_i will be found through the Karush Kuhn-Tucker(KKT) conditions.Let μ_k be the multiplier of the constraint $p_i \leq 1$ and μ_{i+s} is be the multiplier of the constraint $0 \leq p_i$ where $i \in 1, 2, 3..s$. Since the problem is a maximization problem, if (p_i^*, $i \in 1, 2, 3..s$) is an optimal solution to the problem, then (p_i^*, where $i \in 1, 2, 3..s$) must satisfy the following conditions and there must exist multipliers (μ_i and μ_{i+s} where $i \in 1, 2, 3..s$) satisfying the conditions will be discussed below.

Corollary 3.3.1. *Among s shippers, any shipper i with $\frac{r_i}{c_i} > T$ is a full time shipper and shippers having $\frac{r}{c} \leq T$ relation are partial time shippers.*

For a given T, optimal p_i and μ_i values are as in Table 3.2:

Table 3.2: Optimal dual and primal variables, given T

T	p^*	$\mu*$
$\frac{r_s}{c_s} < ... < \frac{r_1}{c_1} \leq T$	$p_i^* = \frac{r_i}{c_i T}, \forall i \in 1, 2, 3..s$	$\mu_i = 0, \forall i \in 1, 2, 3..s.$
$\frac{r_s}{c_s} < ... < \frac{r_{k+1}}{c_{k+1}} \leq T < \frac{r_k}{c_k} < .. < \frac{r_1}{c_1}$.	$p_i^* = 1$ for $1 \leq i < k$ $p_i^* = \frac{r_i}{c_i T}$ for $k \leq i \leq s$	$\mu_i > 0$,$\mu_{i+s} = 0$, for $1 \leq i < k$ $\mu_i = 0$,$\mu_{i+s} = 0$ for $k \leq i \leq s$
$T < \frac{r_s}{c_s} < ... < \frac{r_1}{c_1}$	$p_i^* = 1\ \forall i \in 1, 2, 3..s$	$\mu_i > 0$, $\mu_{i+s} = 0\ \forall i \in 1, 2, 3..s$

Proof. Constraints which are emerged from the KKT conditions are given below:

$$p_i^* = \frac{r_i\lambda_i - \mu_i + \mu_{i+s}}{c_i\lambda_i T} \quad \forall i \in 1, 2, 3..s \tag{3.52}$$

$$\mu_i - \mu_i p_i = 0 \quad \forall i \in 1, 2, 3..s \tag{3.53}$$

$$\mu_{i+s} p_i = 0 \quad \forall i \in 1, 2, 3..s \tag{3.54}$$

$$\mu_i \geq 0 \quad \forall i \in 1, 2, 3..s \tag{3.55}$$

Optimal values are found as follows:

1. When $\mu_i = 0, \forall i$; from Equations 3.52, $p_i^* = \frac{r_i}{c_i T}$ $\forall i$ and $\frac{r_s}{c_s} < ... < \frac{r_1}{c_1} \leq T$ since $0 \leq p_i \leq 1$.

2. When $\mu_i > 0$,$\mu_{i+s} = 0$,from Equations 3.52 and 3.54, $p_i = 1$ for $1 \leq i < k$ and $T < \frac{r_k}{c_k} < ... < \frac{r_1}{c_1}$ since $T = \frac{r_i - \frac{\mu_i}{\lambda_i}}{c_i}$. When $\mu_i = 0$,$\mu_{i+s} = 0$, from Equations 3.52 and 3.54, $p_i^* = \frac{r_i}{c_i T}$ and$\frac{r_s}{c_s} < ... < \frac{r_k + 1}{c_k + 1} \leq T$ for $k \leq i \leq s$ since $0 \leq p_i \leq 1$.

3. When $\mu_i > 0$ and $\mu_{i+s} = 0$ $\forall i$; from Equations 3.53 and 3.52, $p_i^* = 1$ $\forall i$ where $T < \frac{r_s}{c_s} < ... < \frac{r_1}{c_1}$since $T = \frac{r_i - \frac{\mu_i}{\lambda_i}}{c_i}$.

   Following cases have no real or considerable solution. The cases and reasons about why they are not considered are given below:

4. Note that, from Equations 3.53 and 3.54, there will be no solution in any cases where $\mu_i > 0$ and $\mu_{i+s} > 0$ since they can not exist for any corresponding p_i^* values.

5. Note that, from Equations 3.53 and 3.54, there will be no solution in any cases where $\mu_i = 0$ but $\mu_{i+s} > 0$ since p_i^* can not be equal to 0 when r_2,c_2,T and μ_4 have positive values.

■

It can be concluded that among s shippers, shippers having $\frac{r_i}{c_i} > T$ relation are full time shippers and shippers having $\frac{r_i}{c_i} \leq T$ relation are partial time shippers.

According to the findings of p_i^* , total profit of full and partial time shippers at a given T can be expressed as follows:

$$TP(T) = \sum_{i=1}^{k} r_i\lambda_i + \sum_{i=k+1}^{s} \frac{r_i^2\lambda_i}{2c_iT} - (\frac{A}{T} + \sum_{i=1}^{k} \frac{c_i\lambda_iT}{2}) \quad where \quad 0 \le k \le s \quad and$$
$$\frac{r_s}{c_s} < ... < \frac{r_k+1}{c_k+1} \le T < \frac{r_k}{c_k} < ... < \frac{r_1}{c_1} \quad for \quad k \le i \le s. \tag{3.56}$$

In Equation 3.56, k values such that $k \le i \le s$ shows the number of full time shippers among s shippers. When T is fixed; k is already known, thus full time and partial time shippers are known. When T changes, k is determined by the finding the new full time shippers according to comparison of T and $\frac{r}{c}$.

3.3.2 Finding optimal T under optimal p_i

p_i^* values are found and plugged into$TP(T)$.After that, T is the only variable of the total profit function. To maximize total profit value, T^* is determined.

Proposition 3.3.1. $TP(T)$ *function is continuous and differentiable.*

Proof. The function is continuous, since for any T, $\lim_{T \to \frac{r_k}{c_k}^-} TP(T) = \lim_{T \to \frac{r_k}{c_k}^+} TP(T)$.

$$\sum_{i=1}^{k} r_i\lambda_i + \sum_{i=k+1}^{s} \frac{r_i^2\lambda_i}{2c_iT} - (\frac{A}{T} + \sum_{i=1}^{k} \frac{c_i\lambda_iT}{2}) = \sum_{i=1}^{k-1} r_i\lambda_i + \sum_{i=k}^{s} \frac{r_i^2\lambda_i}{2c_iT} - (\frac{A}{T} + \sum_{i=1}^{k-1} \frac{c_i\lambda_iT}{2}) \tag{3.57}$$

When we put $T = \frac{r_k}{c_k}$ for both sides, we obtain the following.

$$r_k\lambda_k = \frac{r_k^2\lambda_kc_k}{2c_k} + \frac{c_k\lambda_kr_k}{2c_k} \tag{3.58}$$

After algebraic cancellations, we obtain the equality of both sides.

To prove the differentiability of $TP(T)$, we will check whether the left and right derivatives of $TP(T)$ with respect to T is equal or not.

$$\frac{\partial}{\partial T}(\sum_{i=1}^{k} r_i\lambda_i + \sum_{i=k+1}^{s} \frac{r_i^2\lambda_i}{2c_iT} - (\frac{A}{T} + \sum_{i=1}^{k} \frac{c_i\lambda_iT}{2})) = \frac{\partial}{\partial T}(\sum_{i=1}^{k-1} r_i\lambda_i + \sum_{i=k}^{s} \frac{r_i^2\lambda_i}{2c_iT} - (\frac{A}{T} + \sum_{i=1}^{k-1} \frac{c_i\lambda_iT}{2})) \tag{3.59}$$

When we put $T = \frac{r_k}{c_k}$ for both sides, we obtain the following.

$$\frac{\partial}{\partial T}(r_k\lambda_k - \frac{c_k\lambda_k r_k}{2c_k}) = \frac{\partial}{\partial T}(\frac{r_k^2\lambda_k}{2c_kT}) \tag{3.60}$$

$$-\frac{c_k\lambda_k}{2} = -\frac{r_k^2\lambda_k c_k^2}{2c_k r_k^2} \tag{3.61}$$

$$-\frac{c_k\lambda_k}{2} = -\frac{c_k\lambda_k}{2} \tag{3.62}$$

The function is differentiable since $\lim_{T\to\frac{r_k}{c_k}^-}\frac{\partial TP(T))}{\partial T} = \lim_{T\to\frac{r_k}{c_k}^+}\frac{\partial TP(T))}{\partial T}$. ■

Proposition 3.3.2. *$TP(T)$ is concave over the region $T \in [0, \overline{T}]$ and non-increasing and convex when $T \in (\overline{T}, \infty)$ for an $\overline{T}$ satisfying $\frac{r_{l+1}}{c_{l+1}} \leq \overline{T} < \frac{r_l}{c_l}$ with $A \leq \sum_{i=l+1}^{s}\frac{r_i^2\lambda_i}{2c_i}$. Here, l can take values from 0 to s.*

Proof. $TP(T)$ equals to $\sum_{i=1}^{l} r_i\lambda_i + \sum_{i=l+1}^{s}\frac{r_i^2\lambda_i}{2c_iT} - (\frac{A}{T} + \sum_{i=1}^{l}\frac{c_i\lambda_iT}{2})$ where $0 \leq l \leq s$ and $\frac{r_s}{c_s} < ... < \frac{r_l+1}{c_l+1} \leq T < \frac{r_l}{c_l} < ... < \frac{r_1}{c_1} \quad for \quad l \leq i \leq s$. To examine increasing/decreasing behavior of the TP(T), the first derivative of $TP(T)$ function is checked :

$$\frac{\partial TP(T)}{\partial T} = (\sum_{i=l+1}^{s}\frac{-r_i^2\lambda_i}{2c_i} + A)(\frac{1}{T^2}) - \sum_{i=1}^{l}\frac{c_i\lambda_i}{2} \tag{3.63}$$

The function is non-increasing when $A \leq \sum_{i=l+1}^{s}\frac{r_i^2\lambda_i}{2c_i}$.

To examine the convex/concave behavior of the TP(T), we check the second derivative of TP(T):

$$\frac{\partial^2 TP}{\partial T^2} = (\sum_{i=l+1}^{s}\frac{r_i^2\lambda_i}{2c_i} - A)(\frac{2}{T^3}) \tag{3.64}$$

So, we can conclude that TP is concave when $\sum_{i=l+1}^{s}\frac{r_i^2\lambda_i}{2c_i} \leq A$, also non-increasing and convex when $A \leq \sum_{i=l+1}^{s}\frac{r_i^2\lambda_i}{2c_i}$. ■

Corollary 3.3.2. *Since $TP(T)$ is continuous and differentiable, there is a unique optimal T value, T^*. Moreover, $T^* \in [0, \overline{T}]$. The reason is that after $\overline{T}$, the function becomes convex and non-increasing as it is shown in Proposition 3.3.2.*

Due to Corollary 3.3.2 , we already know where T^* lies and we have developed the following algorithm to find the region(where a region is $T \in [\frac{r_i}{c_i}, \frac{r_{i-1}}{c_{i-1}}]$) where T^* lies.

Algorithm 1 Algorithm to find the region of optimal T value and the number of partial time shippers

$k \leftarrow s$

while $k \geq 0$ **do**

 if $lim_{T \to \frac{r_k}{c_k}} \frac{\partial TP(T)}{\partial T} \leq 0$ and $lim_{T \to \frac{r_{k+1}}{c_{k+1}}} \frac{\partial TP(T)}{\partial T} \geq 0$ **then**

  $l = k$

  break

 else

  $k \leftarrow l - 1$

 end if

end while

Additionally, it will correspond to the number of partial time shippers in the grand coalition

Hence, $TP(T)$ equals to $\sum_{i=1}^{l} r_i\lambda_i + \sum_{i=l+1}^{s} \frac{r_i^2\lambda_i}{2c_iT} - (\frac{A}{T} + \sum_{i=1}^{l} \frac{c_i\lambda_iT}{2})$ where l is the number founded by the Algorithm 1 and $\frac{r_s}{c_s} < ... < \frac{r_l+1}{c_l+1} \leq T < \frac{r_l}{c_l} < ... < \frac{r_1}{c_1}$. After finding the region of optimal T value and number of partial time shippers, from first order condition of $TP(T)$ and the concavity of the function proven at Proposition 3.3.2, we conclude that $T^* = \sqrt{\frac{2(A - \sum_{i=l+1}^{s} \frac{r_i^2\lambda_i}{2c_i})}{\sum_{i=1}^{l} c_i\lambda_i}}$ by FOC of the TP function. From the equation of T^*, we can express relations of parameters and T^*. If A increases, there will be a T^*. Also, the increase of total profits per unit time made my partial shipper and the increase of full time shippers's total waiting cost at per unit time, $\sum_{i=1}^{l} c_i\lambda_i$, decreases optimal cycle time T^*.

To summarize, we analyze two shipper and s shipper cases according to our problem settings in this chapter. We find the necessary conditions and optimal levels of decision variables such as T and p_i for maximizing total profit for both cases.

CHAPTER 4

THE COOPERATIVE GAME

We found that total profit with joint shipment can be more profitable compared to individual shipping under several conditions. Also, we clarified the optimal shipment interval under several conditions for maximizing total profit of the coalition with s shippers. In this chapter, we work on the problem about finding optimal policy for shippers for participation decisions they made on joint-shipment coalition and allocation of the total profit made between shippers considering concepts of cooperative games.

We study this problem with a cooperative game theoretical approach in this chapter. Firstly, we introduce the game and then investigate the several well known properties of the cooperative games to find that whether our game has these properties or not.

Let v be a real valued function and N be a group of shippers where $v : 2^N \to R$. Given a sub-coalition s such that $s \subseteq N$, let $v(s)$ denote the amount of profit per unit time obtained by s. Note that, $v(s)$, is simply the optimal $TP(T)$ value presented in Proposition 3.3.2 for any coalition. Preceding the findings showing $v(s)$ is higher than total profit value of individual shipments of s shippers, the allocation of $v(s)$ among the shippers is studied.

4.1 The Allocation Scheme for the Cooperative Game

Allocation schemes are critical to set up cooperative games. They define how total profit made by the coalition will be allocated. Without allocation schemes, players will not foresee their profit made by joining coalition and can not make decisions about whether cooperating with the coalition or not.

Now consider the fixed cost of shipment that enables shipper i to operate individually under a cycle length of T^*, where T^* is the optimal cycle length of s shippers shipping jointly. In other words, given $t_i^{'}$ is the optimal cycle length of an individually operating shipper, we are interested in the A value that makes $t_i^{'} = T^*$. Note for shipper i, $t_i^{'} = \sqrt{\frac{2AT_i}{\lambda_i c_i}}$. Then if $A = \frac{(T^*)^2 \lambda_i c_i}{2}$, this will make $t_i^{'} = T^*$. Let the corresponding fixed cost of shipment be denoted with AT_i. Here $AT_i = \frac{(T^*)^2 \lambda_i c_i}{2}$.

In the coalition, there are $s - k$ partial time shippers and k full time shippers. The summation of the AT_i values of the full time shippers is expressed.

$$\sum_{i=1}^{k} AT_i = \sum_{i=1}^{k} \frac{T^{*2} \lambda_i c_i}{2} = \frac{T^{*2} \sum_{i=1}^{k} \lambda_i c_i}{2} \tag{4.1}$$

Meanwhile, note that we can express A as a function of T^* using the expression for T^* under joint shipment:

$$T^* = \sqrt{\frac{2(A - \sum_{i=k+1}^{s} \frac{r_i^2 \lambda_i}{2c_i})}{\sum_{i=1}^{k} c_i \lambda_i}} \tag{4.2}$$

$$A = \frac{T^{*2} \sum_{i=1}^{k} \lambda_i c_i}{2} + \sum_{i=k+1}^{s} \frac{r_i^2 \lambda_i}{2c_i} \tag{4.3}$$

Note also that the expression of A value is the summation of AT_i values and the profit of the partial time shippers in the coalition. This can be stated as in the following equation:

$$A = \sum_{i=1}^{k} AT_i + \sum_{i=k+1}^{s} \frac{r_i^2 \lambda_i}{2c_i} \tag{4.4}$$

We propose a profit allocation scheme with the insights of the equation above: All full time shippers take the same profit as their profit of the individual shipment scenario with AT_i and partial time shippers do not take any profit from coalition. The summation of the allocated profit to full time shippers is equal to total profit gained by the coalition. The equality is reached out by following equations.

We decompose A into terms in Equation 4.4 and then allocate each term with index i to the corresponding shipper. Then we let each shipper to operate individually but optimally under the allocated portion of fixed cost of shipment.

For a full time shipper, optimal profit when operating individually under AT_i would be:

$$r_i\lambda_i - \sqrt{2AT_i\lambda_i c_i} = r_i\lambda_i - \tfrac{AT_i}{T^*} - \tfrac{\lambda_i c_i T^*}{2} = \lambda_i(r_i - c_i T)$$

For a partial time shipper, allocated fixed cost of shipment is $\frac{r_i^2\lambda_i}{2c_i}$. Then corresponding would be $r_i\lambda_i - \sqrt{\frac{2r_i^2\lambda_i\lambda_i c_i}{2c_i}} = 0$.

Note that sum of all profits under allocated A would give $TP(T)$ in Equation 3.56 under T^*.

$$TP(T^*) = \sum_{i=1}^{k} r_i\lambda_i + \sum_{i=k+1}^{s} \frac{r_i^2\lambda_i}{2c_i T} - (\frac{A}{T} + \sum_{i=1}^{k}\frac{c_i\lambda_i T}{2}) = \sum_{i=1}^{k} r_i\lambda_i - (\sum_{i=1}^{k}\frac{AT_i}{T^*} + \sum_{i=1}^{k}\frac{c_i\lambda_i T^*}{2}) \quad (4.5)$$

$$TP(T^*) = \sum_{i=1}^{k} \lambda_i(r_i - c_i T^*) \quad (4.6)$$

Let $ATP_i(T^*)$ denote the allocated profit to full time shipper i.

$$ATP_i = \lambda_i(r_i - c_i T^*) \quad (4.7)$$

In the following, we first discuss some properties of the cooperative shipment consolidation game, before we show that the proposed allocation scheme is in the core.

4.2 Monotonicity of the Cooperative Game

The principle of monotonicity for cooperative games states that if a game changes so that some player's contribution to all coalitions increases or stays the same then the player's allocation should not decrease. (Young, 1985) So, the game is monotonic if the following condition holds:

$$v(S) \leq v(T), for\ \forall S \subset T \subseteq N \quad where\ S\ and\ T\ denote\ subcoalitions \quad (4.8)$$

To investigate the monotonicity of the game, the following results will be used. Let T_s^* denote T^* of the coalition s.

Corollary 4.2.1. *If a new shipper participates in a subcoalition,S, T_S^* decreases.*

Proof. It can be observed from T^* equation:

$$T^* = \sqrt{\frac{2(A - \sum_{i=k+1}^{s} \frac{r_i^2 \lambda_i}{2c_i})}{\sum_{i=1}^{k} c_i \lambda_i}} \tag{4.9}$$

. If the new shipper is full time shipper, denominator of the expression increases and T^* decreases. Otherwise, numerator of the expression decreases and T^* decreases. ■

Corollary 4.2.2. *If a new shipper participates in a subcoalition, S, AT_i and waiting costs decreases .*

Proof. If a new shipper participates in a subcoalition, T^* decreases.(Corollary 4.2.1) When T^* decreases; we can say that AT_i decreases by Equation 4.4 and waiting costs decreases, which equals to $\sum_{i=1}^{l} \frac{c_i \lambda_i T}{2}$. ■

Proposition 4.2.1. *The game is monotonic.*

Proof. If a new shipper participates in a subcoalition, the allocated profit of the full time shipper increase while partial time shippers' allocated amount does not change. Furthermore, a partial time shipper may become a full time shipper. ■

Thus, it is verified that the game is monotonic by Proposition 4.2.1.

4.3 Superadditivity of the Cooperative Game

The profit game is superadditive if sum of profit under two disjoint coalitions is lower than the profit under union of these coalitions:

$$v(S \cup T) \geq v(S) + v(T), \quad for\ all\ disjoint \quad S, T \subseteq N \tag{4.10}$$

Proposition 4.3.1. *In our game, total profit function of a coalition is greater than or equal to summation of the subcoalition's profit functions.*

Proof. Consider set of full time shippers under coalition S and T, each with allocated profit $\lambda_i(r_i - c_i T_S^*)$ or $\lambda_i(r_i - c_i T_T^*)$. In the union of $S \cup T$, profit of each full time shipper would increase. Furthermore, some partial time shippers may become full time shippers and their allocated profit will become positive. Thus, total profit under $S \cup T$ is greater than sum of total profit of S and T. ■

Thus, it is verified that the game is superadditive by Proposition 4.3.1.

4.4 The Core of the Cooperative Game

Let $x = (x_1, x_2, \ldots, x_n)$ be a vector such that the allocation amount for the n^{th} player is x_n. x is called an imputation if it satisfies the group rationality and individual rationality conditions, which are given below. If this imputation also satisfies the group rationality, then it is said that it is in the core.(Winston, 2004) Group rationality guarantees the best allocation amounts for any of players by controlling all possible subcoalitions.

$$v(N) = \sum_{i=1}^{i=n} x_i \quad \text{(Group rationality)} \tag{4.11}$$

$$x_i \geq v(\{i\}) \quad \text{(Individual rationality)} \tag{4.12}$$

If the imputation also satisfies the following condition for $S \subset N$, then it is said that it is in the core.(Winston, 2004)

$$\sum_{i \in S} x_i \geq v(S), \quad S \subset N \tag{4.13}$$

Proposition 4.4.1. *The proposed allocation vector is an imputation.*

Proof. For proving the existence of the proof for the proposed vector, conditions are checked below.

- $v(N)$ is directly equal to the allocation amounts of full time shippers in the coalition and partial time shippers take no profit from the coalition.(Equation 4.6.) So, group rationality condition is satisfied.

- The partial time shippers does not take profit from the coalition but it is not worse than the profit made when they would operate individually, which is zero. The full time shippers increase their allocated profit amount by participating in the coalition. Since they take the profit amount which equals to their individual shipments with with AT_i found in Equation 4.4, instead of A. Since $AT_i < A$, individual rationality condition is also held for the vector.

  Since, individual and group rationality holds, the allocation vector is an imputation.

■

Proposition 4.4.2. *The proposed allocation rule is in the core.*

Proof. In a coalition S, full time shippers get $\lambda_i(r_i - c_i T_s^*)$. As coalition gets bigger in size, each full time shipper get larger profit. Partial time shippers are also get non-decreasing profits. Thus, the proposed allocation is in the core. ■

4.5 Convexity of the Cooperative Game

A cooperative game is convex if the contribution of additional players presents increasing returns to scale.(Shapley,1971)

Shapley(1971) states that a game (N, v) is convex if for all $S, T \in 2^N$ with $S, T \subseteq N$, there is increasing marginal return property. The condition can be expressed via the following inequality:

$$v(S) - v(S \cap T) \leq v(S \cup T) - v(T) \tag{4.14}$$

Equivalently, it can also be expressed as follows for the sets such as $S \subset T \subseteq N$ and $m \notin S, T$:

$$v(T \cup \{m\}) - v(T) \geq v(S \cup \{m\}) - v(T) \tag{4.15}$$

To show the convexity, we use superadditivity of TP function w.r.to λ_i and λ_j.If TP is superadditive, this means that TP increases more with an additional shipper for bigger coalitions.

Lemma 4.5.1. *For any coalition of size s, TP is superadditive.*

Proof. $TP(T) = \sum_{i=1}^{k} r_i\lambda_i + \sum_{i=k+1}^{s} \frac{r_i^2\lambda_i}{2c_iT} - (\frac{A}{T} + \sum_{i=1}^{k} \frac{c_i\lambda_iT}{2})$ where shippers 1 to k are full time shippers and shippers $k+1$ to s are partial time ones. Let set F be the set of full time shippers and set P be the set of partial time shippers. To prove the superadditivity, we check whether the cross partial derivatives of $TP(T)$ with respect to λ_i and λ_j is positive because it shows the rate of change of total profit to shipper i's arrival rate by the increase of shipper j's arrival rate, which can be a change from zero to a positive number.

There are 3 possibilities for partial time or full time shipper distinction for shipper i and shipper j:

- A partial time shipper i and a full time shipper j :

$$\frac{\partial^2 TP}{\partial\lambda_i\partial\lambda_j} = \frac{\frac{r_i^2\lambda_i}{2c_i}}{\sqrt{2(A-\sum_{i=l+1}^{s}\frac{r_i^2\lambda_i}{2c_i})}}\frac{c_j}{c_j\lambda_j+\sum_{k\in F-(j)}c_k\lambda_k} > 0 \tag{4.16}$$

- A partial time shipper $i$and a partial time shipper j, where $\sum_{z=1}^{l} c_z\lambda_z > 0$ as there is at least one full time shipper:

$$\frac{\partial^2 TP}{\partial\lambda_i\partial\lambda_j} = -\sqrt{\sum_{z=1}^{l} c_z\lambda_z}\frac{-\frac{r_i^2\lambda_i}{2c_i}\frac{r_j^2\lambda_j}{2c_j}}{2^{\frac{3}{2}}(A-\frac{r_i^2\lambda_i}{2c_i}-\frac{r_j^2\lambda_j}{2c_j}-\sum_{k\in P-(i,j)}\frac{r_k^2\lambda_k}{2c_k})^{\frac{3}{2}}} > 0 \tag{4.17}$$

- A full time shipper$i$ and a full time shipper j :

$$\frac{\partial^2 TP}{\partial\lambda_i\partial\lambda_j} = \frac{c_ic_j}{4(c_i\lambda_i+c_j\lambda_j+\sum_{k=1}^{l}c_k\lambda_k)^{\frac{3}{2}}}\sqrt{2(A-\sum_{i=l+1}^{s}\frac{r_i^2\lambda_i}{2c_i})} > 0 \tag{4.18}$$

So, the cross partial derivatives are positive and TP is superadditive with respect to λ_i and λ_j. ∎

It is verified that TP is superadditive and bigger coalitions(higherλ rates) leads more increase with new participating shippers. Inequality 4.15, the convexity condition, can also be expressed as follows for an example case where $S \subset T \subseteq N$ and $T = (S \cup \{j\})$:
Let $v(S\cup\{j\}\cup\{m\})$ be the total profit of $S\cup\{j\}\cup\{m\}$. It also equals to $v(T\cup\{m\})$.

$$v(S \cup \{j\} \cup \{m\}) - v(S \cup \{j\}) \overset{?}{\geq} v(T \cup \{m\}) - v(T \cup \{j\})$$
$$\int_0^{\lambda_m} \frac{\partial v(S \cup \{j\} \cup \{m\})}{\partial \lambda_m} d_{\lambda_j} \Big|_{\lambda_j = \lambda_j} \overset{?}{\geq} \int_0^{\lambda_m} \frac{\partial v(S \cup \{j\} \cup \{m\})}{\partial \lambda_m} d_{\lambda_j} \Big|_{\lambda_j = 0}$$
$$\int_0^{\lambda_j} \int_0^{\lambda_m} \frac{\partial v(S \cup \{j\} \cup \{m\})}{\partial \lambda_m \partial \lambda_j} d_{\lambda_j} d_{\lambda_m} \geq 0$$

It is shown that TP is superadditive by verifying that the cross partial derivatives are non-decreasing. So, we find that the above inequality expression for the example case holds and convexity of the profit game is confirmed.

4.6 Example

The following parameters are set for exemplifying the discussed properties of the game. After finding total profits of coalitions, only one example is shared even the properties hold for every possible cases.

Table 4.1: Table of Parameters

Shipper	**r**	**c**	λ	**A**
Shipper 1	105	4	8	
Shipper 2	50	2.9	4	1500
Shipper 3	5	0.8	8	

Under optimality conditions, the total profit values of the coalitions for this problem setting are given in following table:

Table 4.2: TP values and allocations of possible coalitions for the test setting

Coalition $S \in N$	$v(S)$	**Allocations**
{1}	530.16	[530.16]
{2}	13.45	[13.45]
{3}	0	[0]
{1,2}	678.3	[574.6,103.7]
{1,3}	543.4	[543.4,0]
{2,3}	21.4	[21.4,0]
{1,2,3}	693.7	[585.9,107.8,0]

- The condition for monotonicity, Inequality 4.8, holds and it is shown for the following subcoalition:
  $v(\{3, 2, 1\}) >= v(\{3, 2\})$
  $693.7 >= 21.4$

- The condition for superadditivity, Inequality 4.10, holds and it is shown for the following subcoalition:
  $v(\{3, 2, 1\}) >= v(\{3, 2\}) + v(\{1\}$
  $693.7 >= 21.4 + 530.16$

- The conditions for existence of the core; Inequality 4.11,Inequality 4.12 and Inequality 4.13, holds and it is shown for the following subcoalition:
  $v(\{3, 2, 1\}) = x_1 + x_2 + x_3$ (Group Rationality)
  $693.7 = 585.9 + 107.8 + 0$
  $x_1 >= v(\{1\})$ (Individual Rationality)
  $585.9 > 530.16$
  $x_1 + x_2 + x_3 >= v(\{2, 1\})$ (Core condition)
  $693.7 > 678.3$

- The condition for convexity, Inequality 4.15, holds and it is shown for the following subcoalition:
  $v(\{3, 2\} \cup \{1\}) - v(\{3, 2\}) \geq v(\{3\} \cup \{1\}) - v(\{3\})$
  $672.3 \geq 543.3$

CHAPTER 5

NON-COOPERATIVE APPROACH

The shipment consolidation game is also studied as a non-cooperative game where shippers act selfishly. Even if they do joint-shipment, they will decide according to their stand alone profit per unit time regardless of the other shippers profit levels. After we investigate the behavior of shippers under non-cooperative game, we will be able to compare the total profits made by cooperative and non-cooperative games and study on the price of anarchy.

In the following sections, we will discuss our game settings, profit function of shippers and their best response functions. After that, we show that there is a multiple Nash equilibria when there are multiple shippers having the same and smallest cycle time.

5.1 Setting for non-cooperative game

In this game, each shipper needs to decide on contribution level for transportation cost, A. All characteristics of shippers are common knowledge. Each shipper moves simultaneously and determines their contribution level. Then, with the total contribution level, shippers selects the lowest cycle time that can be operated with and shipments occur.

Let C_i be the contribution amount per unit time for player i, $i \in N$. Then $\sum_{i\in N} C_i$ is the total allocated contribution amount in the game. The lowest cycle time is obtained as $T_N(C_1, C_2, ..C_N) = \frac{A}{\sum_{i\in N} C_i}$, which we will shortly show as T_N. Let M be the number of shippers contributing more than 0.

5.2 Profit Function

We write shipper i's profit per unit time as a function of C_i as follows:

$$TP_i(C_i| \sum_{j \in M/\{i\}} C_j) = \begin{cases} r_i\lambda_i - C_i - \frac{c_i\lambda_i A}{2(\sum_{j\in M/\{i\}} C_j + C_i)} & if \quad \frac{r_i}{c_i} > T_N \\ (\frac{r_i^2\lambda_i}{2c_i})(\frac{\sum_{j\in M/\{i\}} C_j + C_i}{A}) - C_i & if \quad \frac{r_i}{c_i} \leq T_N \end{cases} \tag{5.1}$$

The expression follows from the observation that for a given T, if the comparison of $\frac{r_i}{c_i} > T_N$, then shipper i is a full time shipper, otherwise it is a partial time shipper.(see Corollary 3.3.1)

We may equivalantly write the condition $\frac{r_i}{c_i} > T_N$ as a condition on C_i as:

$$\frac{r_i}{c_i} > T_N$$
$$\frac{r_i}{c_i} > \frac{A}{\sum_{i\in M} C_i}$$
$$\frac{r_i}{c_i} > \frac{A}{\sum_{j\in M/\{i\}} C_j + C_i}$$

$$C_i > \frac{Ac_i}{r_i} - \sum_{j\in M/\{i\}} C_j \tag{5.2}$$

Let the right hand side of Inequality 5.2 be denoted with $TH(\sum_{j\in M/\{i\}} C_j)$:

$$TH(\textstyle\sum_{j\in M/\{i\}} C_j) = \frac{Ac_i}{r_i} - \sum_{j\in M/\{i\}} C_j$$

Rewriting Equation 5.1:

$$TP_i(C_i \mid \sum_{j\in M/\{i\}} C_j) = \begin{cases} r_i\lambda_i - C_i - \frac{c_i\lambda_i A}{2(\sum_{j\in M/\{i\}} C_j + C_i)} & if \quad C_i \geq TH(\sum_{j\in M/\{i\}} C_j) \ and \\ (\frac{r_i^2\lambda_i}{2c_i})(\frac{\sum_{j\in M/\{i\}} C_j + C_i}{A}) - C_i & if \quad C_i < TH(\sum_{j\in M/\{i\}} C_j) \end{cases} \tag{5.3}$$

Lemma 5.2.1. *$TP_i(C_i \mid \sum_{j\in M/\{i\}} C_j)$ function is continuous and differentiable in C_i.*

Proof. For continuity, we check whether $\lim_{C_i \to TH(\sum_{j\in M/\{i\}} C_j)^+} TP_i(C_i \mid \sum_{j\in M/\{i\}} C_j) = \lim_{C_i \to TH(\sum_{j\in M/\{i\}} C_j)^-} TP(C_i)$.

$$\lim_{C_i \to TH(\sum_{j\in M/\{i\}} C_j)^+} TP_i(C_i \mid \sum_{j\in M/\{i\}} C_j) = r_i\lambda_i - (\frac{Ac_i}{r_i} - \sum_{j\in M/\{i\}} C_j) - \frac{c_i\lambda_i A}{2(\sum_{j\in M/\{i\}} C_j + \frac{Ac_i - r_i \sum_{j\in M/\{i\}} C_j}{r_i})} \tag{5.4}$$

$$\lim_{C_i \to TH(\sum_{j\in M/\{i\}} C_j)^+} TP_i(C_i \mid \sum_{j\in M/\{i\}} C_j) = \frac{r_i\lambda_i}{2} - (\frac{Ac_i}{r_i} - \sum_{j\in M/\{i\}} C_j) \tag{5.5}$$

$$\lim_{C_i \to TH(\sum_{j\in M/\{i\}} C_j)^-} TP_i(C_i \mid \sum_{j\in M/\{i\}} C_j) = (\frac{r_i^2\lambda_i}{2c_i})(\frac{\sum_{j\in M/\{i\}} C_j + (\frac{Ac_i}{r_i} - \sum_{j\in M/\{i\}} C_j)}{A}) - (\frac{Ac_i}{r_i} - \sum_{j\in M/\{i\}} C_j) \tag{5.6}$$

$$\lim_{C_i \to TH(\sum_{j\in M/\{i\}} C_j)^-} TP_i(C_i \mid \sum_{j\in M/\{i\}} C_j) = \frac{r_i\lambda_i}{2} - (\frac{Ac_i}{r_i} - \sum_{j\in M/\{i\}} C_j) \tag{5.7}$$

For differentiability, we check whether $\lim_{C_i \to TH(\sum_{j\in M/\{i\}} C_j)^+} \frac{\partial TP_i(C_i \mid \sum_{j\in M/\{i\}} C_j)}{\partial C_i} = \lim_{C_i \to TH(\sum_{j\in M/\{i\}} C_j)^-} \frac{\partial TP(C_i)}{\partial C_i}$.

$$\lim_{C_i \to TH(\sum_{j\in M/\{i\}} C_j)^+} \frac{\partial TP_i(C_i \mid \sum_{j\in M/\{i\}} C_j)}{\partial C_i} = -1 + \frac{c_i\lambda_i A}{2(\sum_{j\in M/\{i\}} C_j + (\frac{Ac_i}{r_i} - \sum_{j\in M/\{i\}} C_j)^+)^2} \tag{5.8}$$

$$\lim_{C_i \to TH(\sum_{j\in M/\{i\}} C_j)^+} \frac{\partial TP_i(C_i \mid \sum_{j\in M/\{i\}} C_j)}{\partial C_i} = -1 + \frac{r_i^2\lambda_i}{2Ac_i} \tag{5.9}$$

$$\lim_{C_i \to TH(\sum_{j\in M/\{i\}} C_j)^-} \frac{\partial TP(C_i)}{\partial C_i} = -1 + \frac{r_i^2\lambda_i}{2Ac_i} \tag{5.10}$$

So, we prove that $TP_i(C_i \mid \sum_{j\in M/\{i\}} C_j)$ function is continuous and differentiable. ■

5.3 Characterization of Nash Equilibria

In this part, we first find the best response function and then characterize the Nash equilibria.

Let $r_i(\sum_{j\in M/\{i\}} C_j)$ be the best response function of the shipper i in terms of contribution amount of shipper i and given total contribution of other shippers.

Since each player wants to maximize his/her profit, the best response function can be expressed as follows: $r_i(\sum_{j\in M/\{i\}} C_j) = \underset{C_i}{\operatorname{argmax}}\ TP_i(C_i \mid \sum_{j\in M/\{i\}} C_j)$

Proposition 5.3.1. *Best response function can be characterized as follows:*

$$r_i(\sum_{j\in M/\{i\}} C_j) = \begin{cases} \sqrt{\frac{c_i\lambda_i A}{2}} - \sum_{j\in M/\{i\}} C_j & if \quad \frac{r_i^2\lambda_i}{2Ac_i} \geq 1 \; and \; \sqrt{\frac{c_i\lambda_i A}{2}} \geq \sum_{j\in M/\{i\}} C_j \\ 0 & if \quad \frac{r_i^2\lambda_i}{2Ac_i} \geq 1 \; and \; \sqrt{\frac{c_i\lambda_i A}{2}} < \sum_{j\in M/\{i\}} C_j \\ 0 & if \quad \frac{r_i^2\lambda_i}{2Ac_i} < 1 \end{cases} \tag{5.11}$$

Proof. We characterize best responses for possible values of $TH(\sum_{j\in M/\{i\}} C_j)$.

1. $TH(\sum_{j\in M/\{i\}} C_j) \geq C_i$:

   We checked the first order condition of $TP(C_i)$, when $TH(\sum_{j\in M/\{i\}} C_j) \geq C_i$:

   $$\frac{\partial TP(C_i)}{\partial C_i} = -1 + \frac{r_i^2\lambda_i}{2Ac_i} \ .$$

   The first derivative of $TP(C_i)$ indicates that the slope of the function is constant and the function changes linearly with C_i when $TH(\sum_{j\in M/\{i\}} C_j) \geq C_i$.

   When $\frac{r_i^2\lambda_i}{2Ac_i} \geq 1$, the slope of $TP(C_i)$ is positive, thus $C_i^* = TH$ and we need to search global optimal solution at $C_i > TH$ region. (Note that TP function is continuous and differentiable.)

   When $\frac{r_i^2\lambda_i}{2Ac_i} < 1$, slope of the function is negative, so we need to search optimal solution at smallest value in that region. $C_i^* = 0$ is the best response value in that region.

2. When $TH(\sum_{j\in M/\{i\}} C_j) < C_i$:

When $\frac{r_i^2\lambda_i}{2Ac_i} \geq 1$, slope of $TP(C_i)$ is positive, so we need to search global optimal solution at $C_i > TH$ region as it was proved that TP function is continuous and differentiable.The first order condition of $TP(C_i)$ when $C_i > TH$ as follows:

$$\frac{\partial TP(C_i)}{\partial C_i} = -1 + \frac{c_i\lambda_i A}{2(\sum_{j\in M/\{i\}} C_j + C_i)^2} .$$

From the first order condition of $TP(C_i)$, $C_i^* = \sqrt{\frac{c_i\lambda_i A}{2}} - \sum_{j\in M/\{i\}} C_j$ when $\sqrt{\frac{c_i\lambda_i A}{2}} \geq \sum_{j\in M/\{i\}} C_j$, $C_i^* = \sqrt{\frac{c_i\lambda_i A}{2}} - \sum_{j\in M/\{i\}} C_j$. C_i^* needs to be within the range C_i. Since $C_i^* \geq 0$, $C_i^* = 0$ when $\sqrt{\frac{c_i\lambda_i A}{2}} < \sum_{j\in M/\{i\}} C_j$.

When $\frac{r_i^2\lambda_i}{2Ac_i} < 1$, slope of the function is negative, so we need to search optimal solution at smallest value of the C_i^* region. Since C_i^* needs to be greater than or equal to 0. $C_i^* = 0$.

Note that TH value is key for differing to become full time or partial time shipper case. In determining the best response, the boundaries of C_i is more decisive for the value of C^*. ■

Observation. *Note that there is an upper bound for the contribution level that shipper i can do. C_{imax} equals to $\sqrt{\frac{c_i\lambda_i A}{2}}$ when $\sum_{j\in M/\{i\}} C_j = 0$. This levels equals to $\frac{A}{t_i^*}$ where t_i^* is $\frac{2A}{c_i\lambda_1}$. In other words, that is the optimal level shipper i pay in their individual shipment case showing that there will be no shippers paying more than the amount to reach out t_i^*.*

Observation. *Note that if $\frac{r_i^2\lambda_i}{2Ac_i} \geq 1$, shipper i is self-sufficient to operate individually and if $\frac{r_i^2\lambda_i}{2Ac_i} < 1$, shipper i is not self-sufficient to do shipments. These are validated from individual shipment total profit function, $r_i\lambda_i - \sqrt{2Ac_i\lambda_i}$. (see 3.3)*

Corollary 5.3.1. *There are no partial time shippers who are making any contribution a non-cooperative game. Shippers are whether free-riders or contributors as full time shippers.*

When shippers act selfishly, it is meaningful that they will not contribute for the transportation cost where they can participate partially only. Not self-sufficient shippers are not able to do individual shipments, so there is no reason for them to contribute for joint-shipment. If they are self-sufficient, they check their t^* value and for smaller cycle times they will not prefer to contribute.

Next, we characterize the Nash Equilibria. Let n be the number of self-sufficient shippers that are able to make shipments individually and m be the number of shippers that have the smallest optimal cycle time for individual shipments,t_i^* among N shippers, (possibly $m \geq 2$ and $m \leq n \leq s$). In Lemma 3.0.1, it is expressed that $t_i^* = \sqrt{\frac{2A}{c_i \lambda_i}}$. Let w.l.o.g. suppose that $t_1^* = t_2^* = .. = t_m^* < t_{m+1}^* \leq ... \leq t_n^*$. Equivalently, $c_1\lambda_1 = c_2\lambda_2 ... c_m\lambda_m > c_{m+1}\lambda_{m+1} \geq ... \geq c_n\lambda_n$

Theorem 5.3.1. *For $n > 0$, following contributions form multiple Nash equilibria:*

$(C_1^*, C_2^*..C_m^*) = \{x_1, x_2, ..x_m : x_i \geq 0, \sum_{i=1}^m x_i = \sqrt{\frac{c_1\lambda_1 A}{2}}\}, \quad 1 \leq j \leq m, \quad m = 1, 2, ..n$ *and*

$C_j = 0, \quad j \geq m+1.$

For $n = 0$, following strategy profile is the unique Nash equilibria: $C_i^* = 0 \;\; i \in N$.

Proof. For all m shippers, best response will be $\sqrt{\frac{c_i\lambda_i A}{2}} - \sum_{j \in M/\{i\}} C_j$, since $\frac{r_i^2 \lambda_i}{2Ac_i} \geq 1$ for all $i \in m$. (Equation 5.11). Since their $c_i\lambda_i$ values are equal; summation of their contribution levels will be equal to $\sqrt{\frac{c_1\lambda_1 A}{2}}$. Thus, C_i for all $i \in m$ will become as follows:

$$\begin{aligned}
C_1 &= \sqrt{\tfrac{c_1\lambda_1 A}{2}} - \textstyle\sum_{j\in[1,m]/\{1\}} C_j \\
C_2 &= \sqrt{\tfrac{c_1\lambda_1 A}{2}} - \textstyle\sum_{j\in[1,m]/\{2\}} C_j \\
C_i &= \sqrt{\tfrac{c_1\lambda_1 A}{2}} - \textstyle\sum_{j\in[1,m]/\{i\}} C_j \\
C_m &= \sqrt{\tfrac{c_1\lambda_1 A}{2}} - \textstyle\sum_{j\in[1,m]/\{m\}} C_j
\end{aligned}$$

Summing C_i for all $i \in m$'s gives us:

$$\textstyle\sum_{j\in[1,m]} C_j = m\sqrt{\tfrac{c_1\lambda_1 A}{2}} - (m-1)\textstyle\sum_{j\in[1,m]/\{m\}} C_j$$

After, m's cancel out, it is found that:

$$\sum_{j\in[1,m]} C_j = \sqrt{\frac{c_1\lambda_1 A}{2}} \tag{5.12}$$

So, for all m shippers, contribution levels that are $\sum_{j\in[1,m]} C_j = \sqrt{\frac{c_1\lambda_1 A}{2}}$ are multiple equilibrium points.

Recall the observation about self sufficient shippers having $\frac{r_i^2\lambda_i}{2Ac_i} \geq 1$. Since all n shippers are self-sufficient, they have $\frac{r_i^2\lambda_i}{2Ac_i} \geq 1$. But there will be no more than one shipper group, that has same t_i^*, willing to contribute $\sqrt{\frac{c_i\lambda_i A}{2}} - \sum_{j\in M/\{i\}} C_j$ amount according to Equation 5.11.

In the following, we show that when $m = 1$, shipper 1 is the only shipper with $C_1 > 0$ and all other shippers must have $C_i = 0$. The proof is done by contradiction:

Suppose there are two self-sufficient shippers with $C_i > 0 : C_1 > 0, C_2 > 0$ and $t_1^* < t_2^*$. C_1 equals to $\sqrt{\frac{c_1\lambda_1 A}{2}} - C_2$ and C_2 equals to $\sqrt{\frac{c_2\lambda_2 A}{2}} - C_1$ according to Equation 5.11 assuming for now that $\sqrt{\frac{c_1\lambda_1 A}{2}} \geq C_2$ and $\sqrt{\frac{c_2\lambda_2 A}{2}} \geq C_1$.

Plugging C_2 into C_1 equation gives us:

$$C_1 = \sqrt{\frac{c_1\lambda_1 A}{2}} - \left(\sqrt{\frac{c_2\lambda_2 A}{2}} - C_1\right)$$

$$C_1 = \frac{\sqrt{\frac{c_1\lambda_1 A}{2}}}{2} - \frac{\sqrt{\frac{c_2\lambda_2 A}{2}}}{2}$$

Plugging C_2 into C_1 equation gives us:

$$C_2 = \sqrt{\frac{c_2\lambda_2 A}{2}} - \frac{\sqrt{\frac{c_1\lambda_1 A}{2}}}{2} - \frac{\sqrt{\frac{c_2\lambda_2 A}{2}}}{2}$$

$$C_2 = \frac{\sqrt{\frac{c_2\lambda_2 A}{2}}}{2} - \frac{\sqrt{\frac{c_1\lambda_1 A}{2}}}{2}$$

It is well known that $t_i^* = \sqrt{\frac{2A}{c_i\lambda_i}}$.since we assume that $t_1^* < t_2^*$;

$$\sqrt{\frac{2A}{c_1\lambda_1}} < \sqrt{\frac{2A}{c_1\lambda_1}}$$

Reforming this to relate with C_2:

$$\frac{A}{t_1^*} = \sqrt{\frac{c_1\lambda_1 A}{2}} > \frac{A}{t_2^*} = \sqrt{\frac{c_2\lambda_2 A}{2}}$$
$$C_2 = \frac{\sqrt{\frac{c_2\lambda_2 A}{2}}}{2} - \frac{\sqrt{\frac{c_1\lambda_1 A}{2}}}{2} < 0$$

This conflicts with our starting assumption that $C_2 > 0$. Thus, there can not be 2 players contributing positive amount under Nash equilibria with $m = 1$.

Secondly, there is no equilibrium point where shipper 1 is not the one who is contributing . Only shipper 1 contributes with $C_1 = \sqrt{\frac{c_1\lambda_1 A}{2}}$. To show the reason behind, suppose shipper 2 with $t_1 \leq t_2$ has $C_2 > 0$ and all of the shippers have $C_i = 0$. Specifically, we have $\sqrt{\frac{c_2\lambda_2 A}{2}} \geq \sum_{j \in M/\{2\}} C_j$, $C_2 = \sqrt{\frac{c_2\lambda_2 A}{2}} - \sum_{j \in M/\{i\}} C_j$ and $c_2\lambda_2 < c_1\lambda_1$. Since, $\sqrt{\frac{c_1\lambda_1 A}{2}} \geq \sqrt{\frac{c_2\lambda_2 A}{2}} \geq \sum_{j \in M/\{i\}} C_j$ from best response function $C_1 = \sqrt{\frac{c_2\lambda_2 A}{2}} - \sum_{j=3,4..} C_j > 0$, which contradicts the assumption that $C_2 > 0$, $C_i = 0, i \neq$

When $m = 0$, none of the shippers are self-sufficient, i.e., $\frac{r_i^2\lambda_i}{2Ac_i} < 1$. Their best response is not to contribute and none of them will want to deviate from the equilibrium since there is no profit to take with other actions.

■

For a 2 shipper coalition, best response graph is analyzed. As it can be seen Figure 5.1 also, both shipper will not deviate when shipper 1 contribute the amount needed to reach out their stand-alone shipment cycle time and shipper 2 will not contribute. The parameter sets are $(r_1, c_1, \lambda_1) = (105, 4, 8)$,$(r_2, c_2, \lambda_2) = (50, 2.9, 4)$ and $A = 1300$.

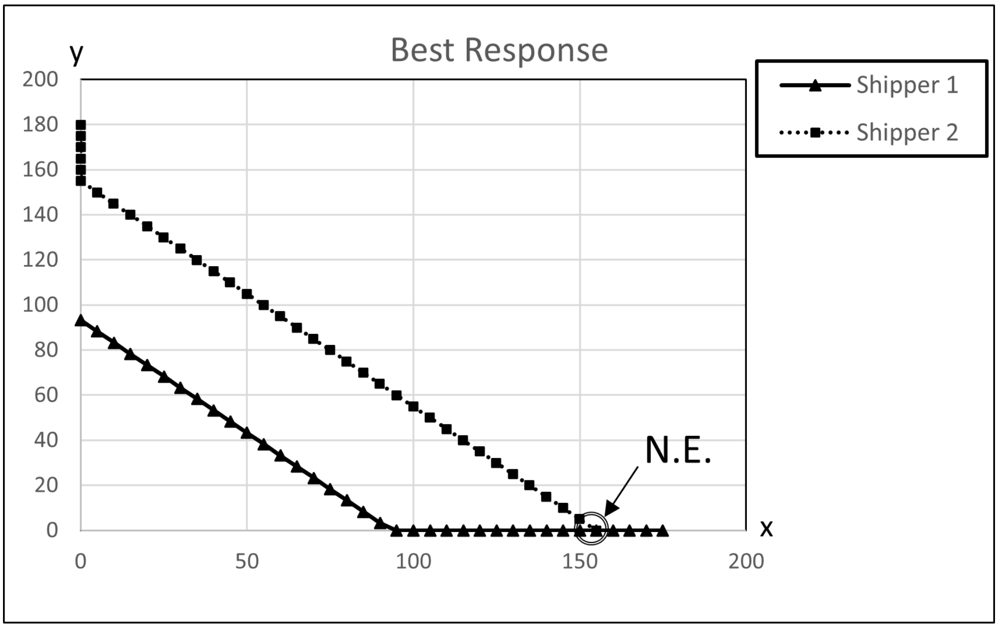

Figure 5.1: A Graph of contribution levels of 2 self-sufficient shipper, $t_1^* < t_2^*$

CHAPTER 6

NUMERICAL ANALYSIS

In this chapter, we provide numerical analysis on cooperative and non-cooperative scenarios for shipper groups having different characteristics and aim to quantify the price of anarchy by comparing the profit of each shipper. The price of anarchy is defined by the ratio of the profit difference under cooperative game and non-cooperative game over the profit made by the cooperative game. The higher value of price of anarchy shows more profit loss due to selfish attitudes of the shippers. We expect values between zero and one. In the proposed allocation scheme of the cooperative game, the partial shippers take 0 profit. For these cases, we take price of anarchy as zero since there can be no efficiency loss due to selfish behaviors of shippers.

Additionally, we also calculate Gini coefficient under cooperative and non-cooperative games to observe the profit allocation differences among shippers. Gini coefficient is defined as the mean difference from all observed quantities (Gini, 1912). It is a measure of inequality of a distrubition which takes value between 0 and 1. Zero represents perfect profit equality and one represents perfect profit inequality. The coefficient is found by the ratio of the area between the Lorenz Curve of the distribution and the uniform distribution line and the area under the uniform distribution line. The Lorenz Curve is developed to demonstrate income distributions by Lorenz (1905). It shows the proportion of total income is in the hands of a given percentage of population. An example of The Lorenz Curve for a non-cooperative game with 5 shippers is calculated. It can be seen in Figure 6.1.

In our analysis, since we have results for discrete levels instead of continuous ones, we approximate the Gini coefficient of each scenario by dividing summation of the differences of cumulative perfect equal profit allocation from cumulative profit allocated

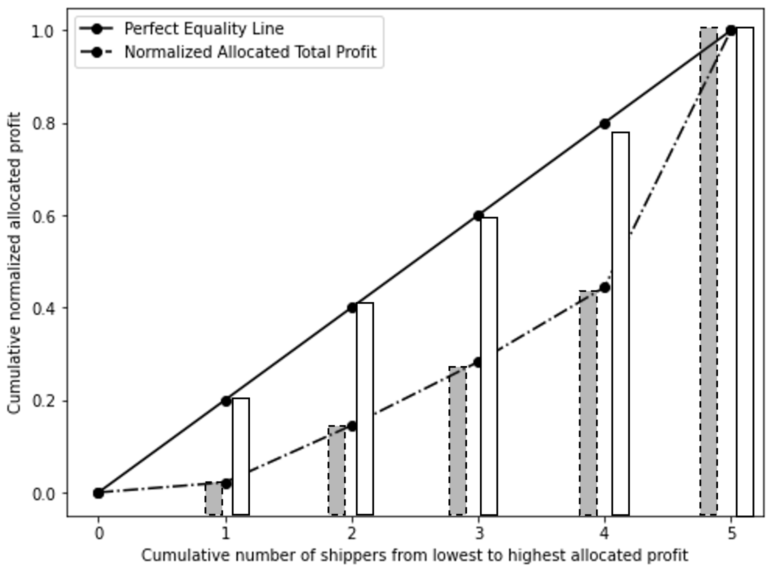

Figure 6.1: The Lorenz Curve and Perfect Equality Line for a non-cooperative profit game

under Lorenz curve to summation of the cumulative profit under perfect distribution rule. We normalize the allocated profits and make an ascending order of shipper's allocated profit. Normalization is made according to ratio of cumulative allocated profit at for the number of shipper at that point to total profit made by all shippers. In Figure 6.1, Lorenz curve of a non-cooperative game and the perfect equality line is exemplified. Here, the Gini coefficient is found by differences of sum of white boxes from sum of grey boxes divided by sum of white boxes. Lorenz curve and Gini coefficient is calculated for both cooperative and non-cooperative games to compare the allocation fairness.

For the calculations of total profit under cooperative games, $TP(T^*)$ function (Equation 4.6) is computed where $T^* = \sqrt{\frac{2(A - \sum_{i=l+1}^{s} \frac{r_i^2 \lambda_i}{2c_i})}{\sum_{i=1}^{l} c_i \lambda_i}}$ where l is the full time shipper in the coalition and s is the number of shippers in coalition. For each shipper's profit comparison, we consider our allocation scheme: Full time shippers take $\lambda_i(r_i - Tc_i)$ where $i \in [0, l]$ and partial time shippers do not take any profit. This profit amount equals to the same profit they make under individual shipment scenario with AT_i transportation cost.

For the calculations of total profit under non-cooperative games, by Theorem 5.3.1, we find the shipper group, $m \geq 1$, having minimum optimal cycle time at individual shipment case, t_1^*, and determine the total contribution level: $\sum_{j \in m} C_j = \sqrt{\frac{c_1 \lambda_1 A}{2}}$. Then, $TP_i(C_i | \sum_{j \in M/\{i\}} C_j)$ function at Equation 5.1 is computed for each shipper i, considering their TH values.

For both games, we also check average individual profits of shippers and price of anarchy. This is calculated by total profits under a coalition divided by number of shippers in the coalition, N. This shows the changes of the allocation in both games.

For testing a wide variety of cases, we study on different levels of N, A,r, c, λ. We create a base case for parameters (Table 6.1):

Table 6.1: Parameter values corresponding to base case

Parameter	**Value/Range**
N	5
A	100
r	U[5,30]
λ	U[2,10]
c	U[2,7]

For each parameter, there are different levels to be tested. While testing a parameter, the other independent parameters remain at the base case.

Table 6.2: Levels of parameter values

Parameter	**Value/Range**
N	3, 5, 10, 20
A	10, 50, 100, 150, 200, 300
r	U[5,40], U[20,50], U[30,70]
λ	U[2,10], U[15,25]
c	U[0,1], U[1,5], U[5,10], U[10,15]

In the following sections, the scenarios are created for measuring the effects of the parameters on our performance measures, which are average individual profit levels, total profit levels, Price of Anarchy (PoA) and Gini coefficients for cooperative and non-cooperative game. Each parameter will be discussed separately. For each test scenario, we make 5 replications and take average of them to eliminate the outlier cases coming from randomness.

6.1 Effect of Number of Shippers

For observing the effect of number of shippers in the coalitions, we check a variety of N values, which can be seen in 6.2. Starting with average individual profits, it is lightly increasing or almost remaining same with respect to increase in N as seen in Figure 6.2. Price of anarchy is slightly decreasing as there can be more free shippers making profits in non-cooperative game whereas there can be more partial shippers who takes no profit in cooperative game. Total profits made under both cooperative

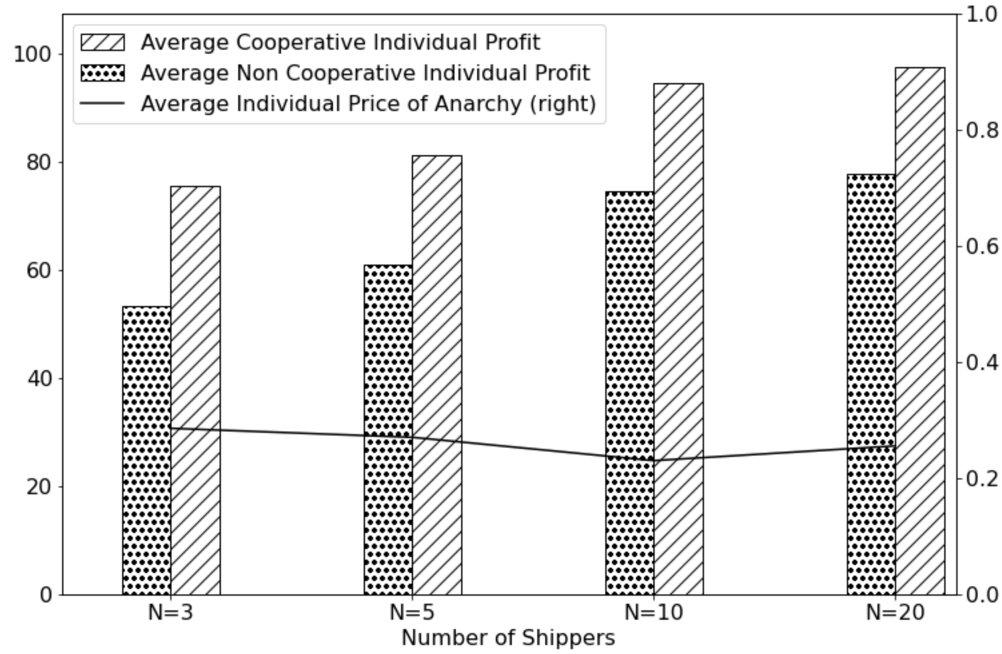

Figure 6.2: Change in average individual profit under cooperative and non-cooperative games and change in average individual PoA with respect to a change in N

and non-cooperative game increases with N since more shippers make more profits. The price of anarchy slightly decreases with the same reason in average individual profit case. (Figure 6.3) The Gini coefficient under cooperative and non-cooperative

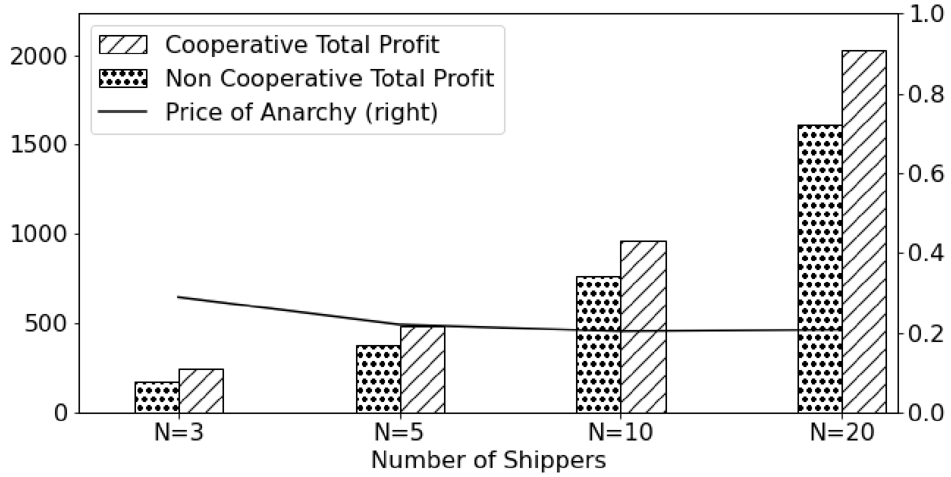

Figure 6.3: Change in total profit and price of anarchy under cooperative and non-cooperative games with respect to a change in N

game shows variance at different N levels.(Figure 6.4) When number of shippers increases, allocation in non-cooperative game seems more fair according to Gini coefficient since the coefficient is lower under non-cooperative games at higher number of shippers.

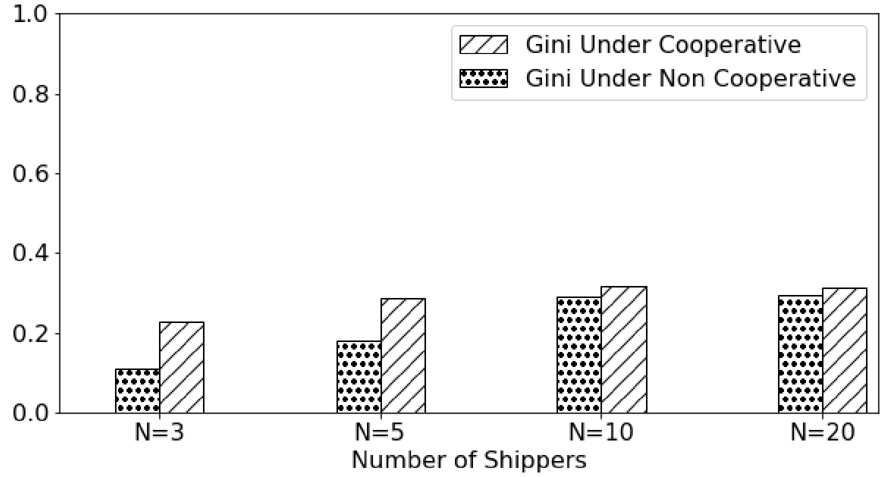

Figure 6.4: Gini coefficient under cooperative and non-cooperative games with respect to a change in N

6.2 Effect of Transportation Cost

For observing the effect of transportation cost in the coalitions, we check a variety of A values, which can be seen in 6.2. Average individual profits are decreasing with respect to increase in A since the coalition needs to pay higher cost for making shipments.(Figure 6.5). Price of anarchy is consistently increasing with A, which implies that in higher transportation cost, inefficiency of selfishness increasing and shippers should collaborate for more profits. The reason is that in non-cooperative case, the transportation cost is paid by not the whole coalition but a small group or just a shipper having smallest individual cycle time t_i^* pays all of the transportation cost and the contribution is just enough to make same profit with invidiual shipment case for the contributor shipper. Since the contributor shipper's individual profitability is decreasing with respect to increase in A and the other shippers are free shippers, they can not reach higher profit levels.

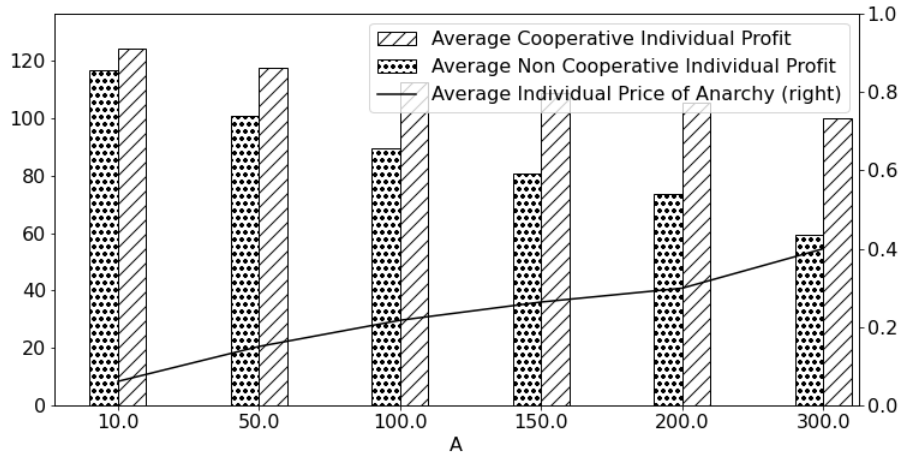

Figure 6.5: Change in average individual profit and price of anarchy under cooperative and non-cooperative games with respect to a change in transportation cost, A

Total profits of cooperative and non-cooperative games are decreasing and price of anarchy is increasing with A that can be seen in Figure 6.6.

Gini coefficient under non-cooperative game is lower than under cooperative game for all transportation cost levels. That is because non-cooperative game has less total profits and free shippers.

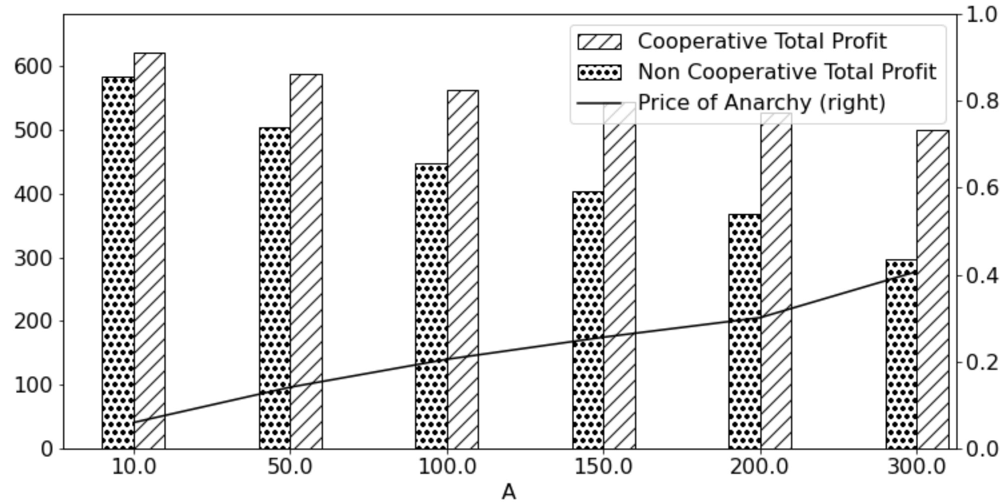

Figure 6.6: Change in total profit and price of anarchy under cooperative and non-cooperative games with respect to a change in transportation cost, A

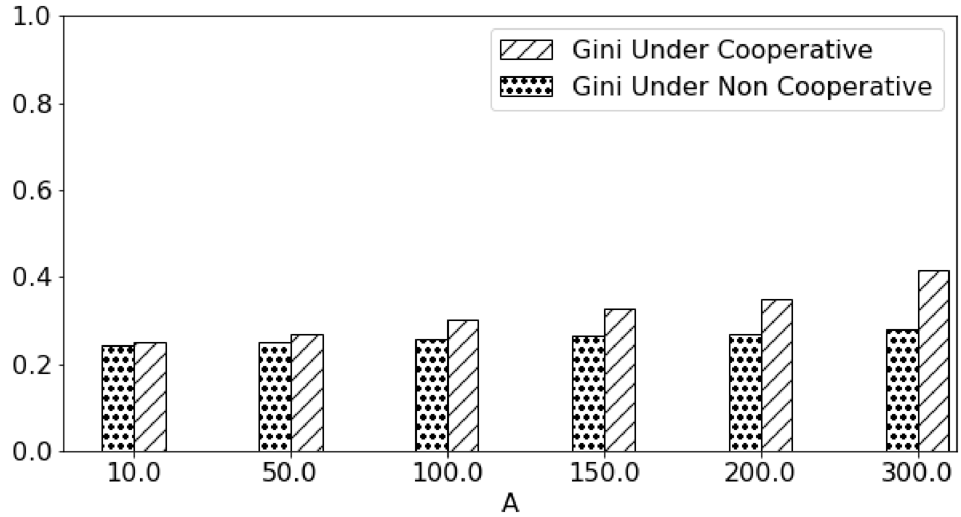

Figure 6.7: Change in Gini coefficient under cooperative and non-cooperative games with respect to a change in transportation cost, A

6.3 Effect of Revenue

For observing the effect of transportation cost in the coalitions, we check a variety of r values, which can be seen in 6.2. Firstly, average individual profits are increasing with respect to increase in r since the coalition takes higher revenue for unit of shipment(Figure 6.8). Price of anarchy is consistently decreasing with r, which implies that in higher per unit revenue, inefficiency of selfishness is decreasing. As it is discussed, in non-cooperative case, the transportation cost is paid by not the whole coalition but a small group or just a shipper having smallest individual cycle time t_i^* at that transportation cost pays all of the cost. The group contributor is willing to contribute more to take profits at least at the same level of individual shipment case which leads to more profits for the partial shipper group under the non-cooperative game.

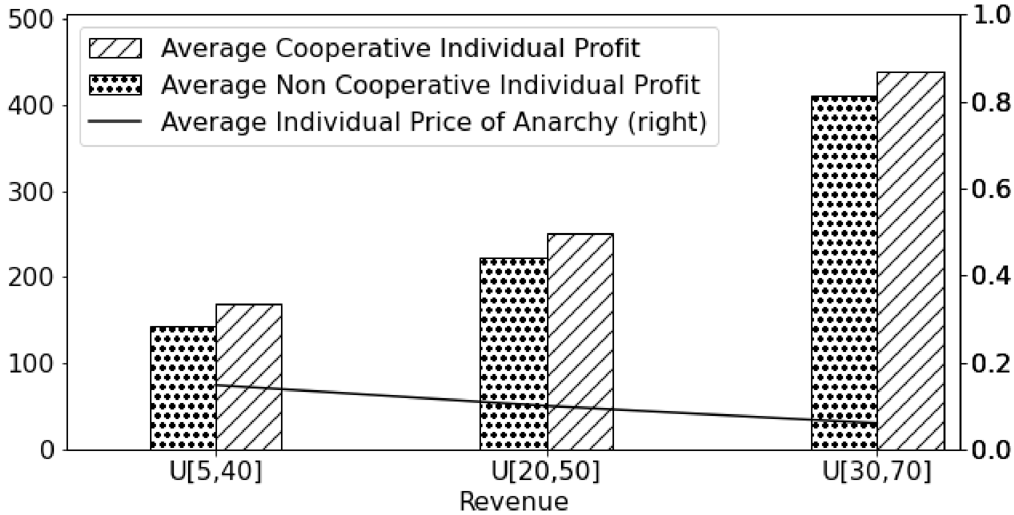

Figure 6.8: Change in average individual profit and price of anarchy under cooperative and non-cooperative games with respect to a change in r

Change in total profit under cooperative and non-cooperative games is similar to change in average individual pattern. Profits are increasing but total profits under cooperative game are higher than total profits under non-cooperative game. Price of anarchy is also decreasing by increase of revenue because the free shippers starting to make more profits in non-cooperative game while full time shippers in the cooperative game takes profits by paying AT_i values.

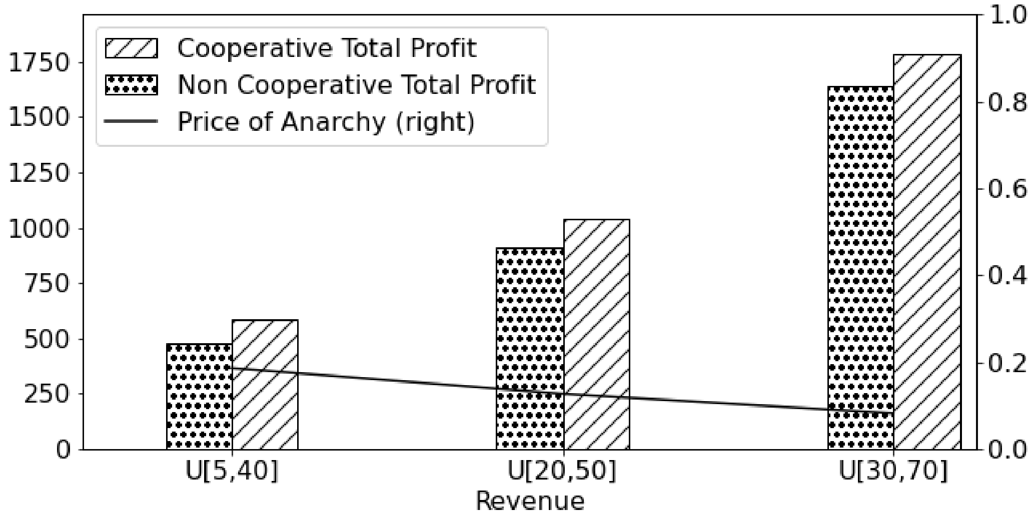

Figure 6.9: Change in total profit and price of anarchy under cooperative and non-cooperative games with respect to a change in r

Gini coefficient does not get affected by r.

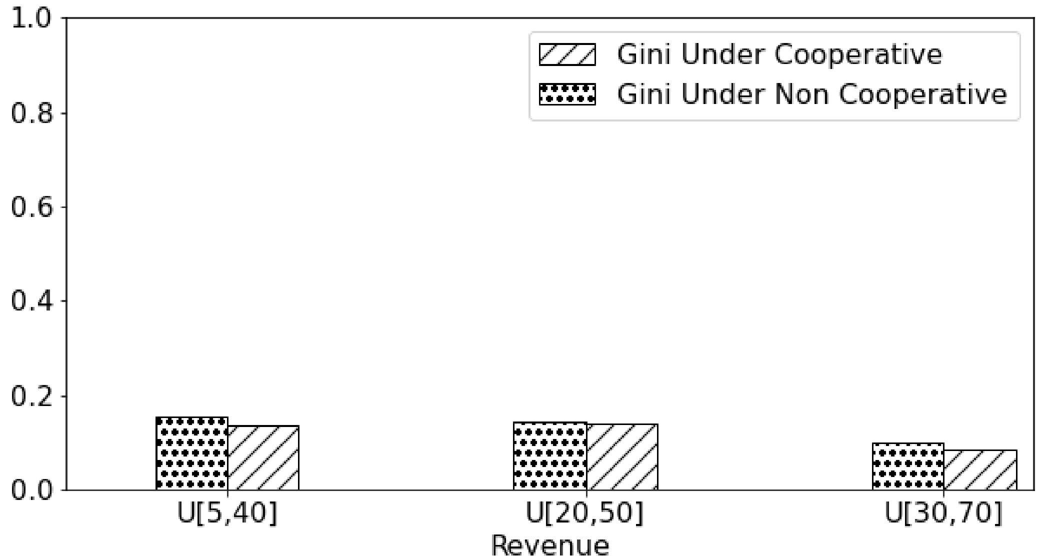

Figure 6.10: Gini coefficient under cooperative and non-cooperative games with respect to a change in r

6.4 Effect of Waiting Cost

For observing the effect of waiting cost in the coalitions, we check a variety of c values, which can be seen in Table 6.2. Average individual profits reaches their maximum levels when the cost is at minimum uniform distribution range which is U[0,1]. Then, the average individual profits made under the games tends to decrease with increase of waiting cost, c. The reason is that increase in waiting cost is independently decreasing the profit made under both games which can be seen from Equation 5.1 and 4.7.

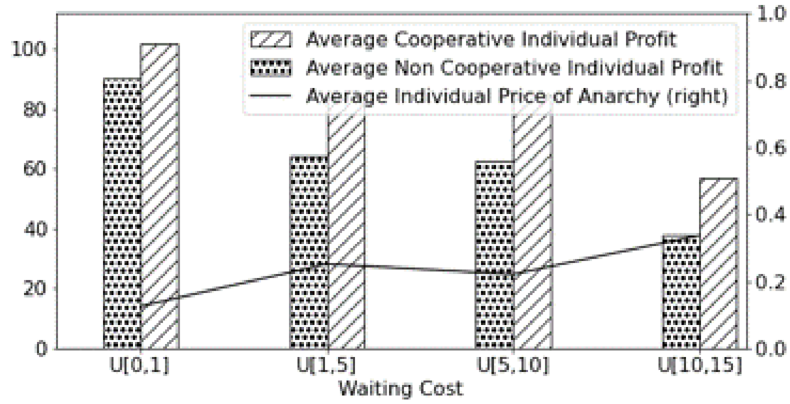

Figure 6.11: Change in average individual profit and price of anarchy under cooperative and non-cooperative games with respect to a change in c

Total profit under cooperative and non-cooperative games consistently decreases with the increase of waiting cost parameter (Figure 6.12). Also, the price of anarchy increases since, as it is discussed, in the non-cooperative case, the transportation cost is paid by not the whole coalition but a small group or just a shipper having the smallest individual cycle time t_i^* pays all of the transportation cost which causes to profit loss. Since the group contributor becoming less eligible to pay more and there is no more contributor, shippers can not make more frequent shipment and take less profits comparing to the cooperative game.

The Gini coefficient results indicates that under increase of waiting cost, total profits made under both games consistently decreasing and after a certain level the allocation becomes more unfair in cooperative game because the partial shipper number increases.

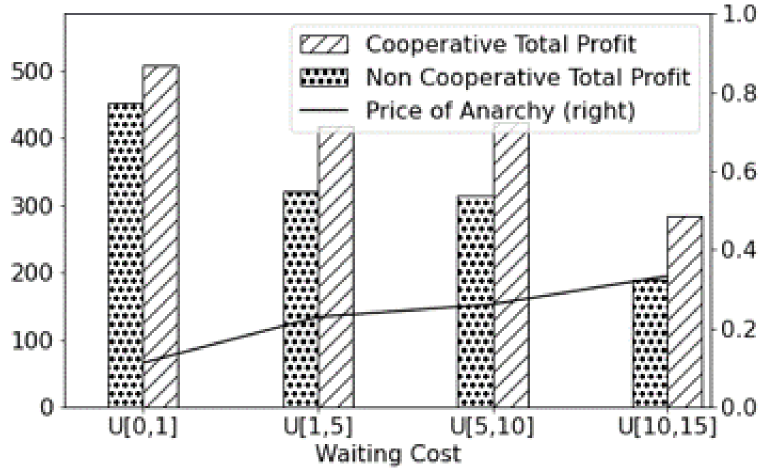

Figure 6.12: Change in total profit and price of anarchy under cooperative and non-cooperative games with respect to a change in c

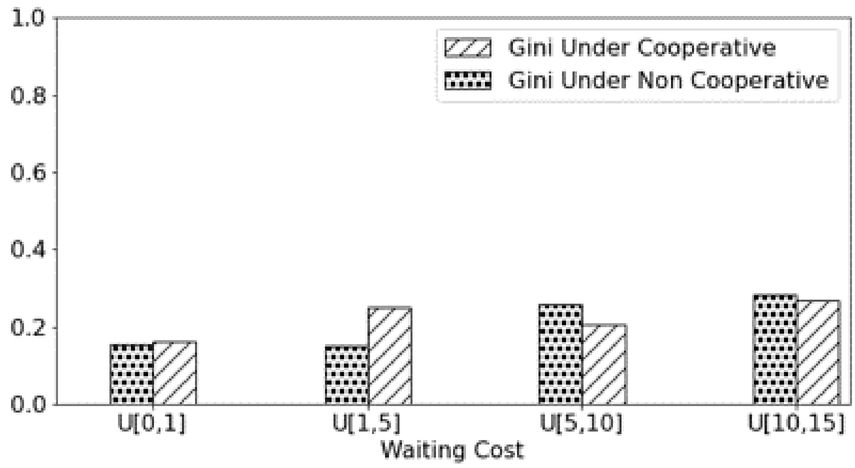

Figure 6.13: Change in Gini coefficient under cooperative and non-cooperative games with respect to a change in c

6.5 Effect of Arrival Rate

For observing the effect of arrival rate in the coalitions, we check a variety of λ values, which can be seen in 6.2. From Figure 6.14, with the increase of arrival rates, average of individual profits under both games increases. In both arrival rate levels, the average individual profit under non-cooperative game is less than the profit under cooperative game.

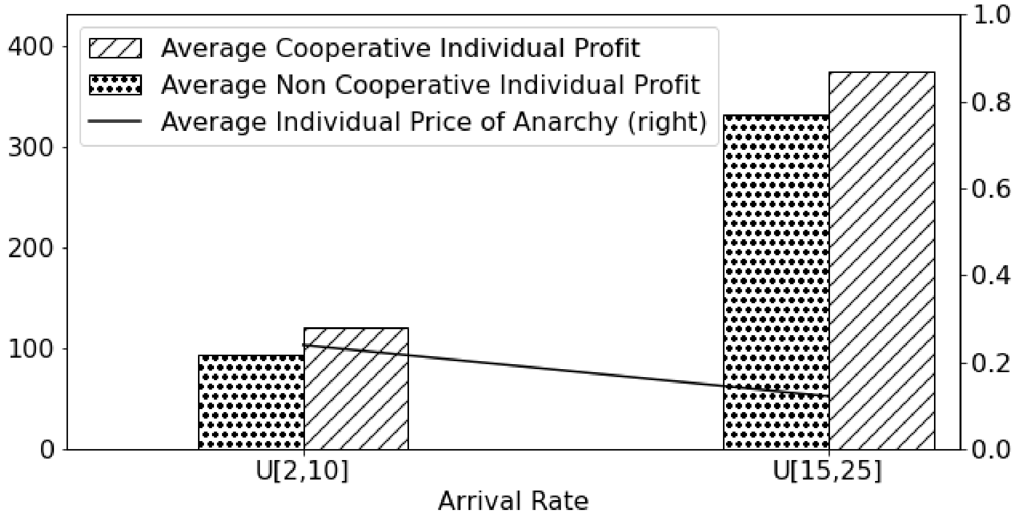

Figure 6.14: Change in average individual profit and price of anarchy under cooperative and non-cooperative games with respect to a change in λ

From Figure 6.15, with the increase of arrival rates, total profits under both games increases. In both tested arrival rate levels, the total profit under non-cooperative game is less than the profit under cooperative game. It is observed that change in the price of anarchy of the average individual profits is similar to change in the price of anarchy of total profits.

It can be observed from Figure 6.16 that with the increase of arrival rate slightly decreases the Gini coefficient which means the difference of allocated profits of shippers slightly decreases. Also, there is not a significant difference of the Gini coefficient under cooperative and non-cooperative games but they are not equal to each other.

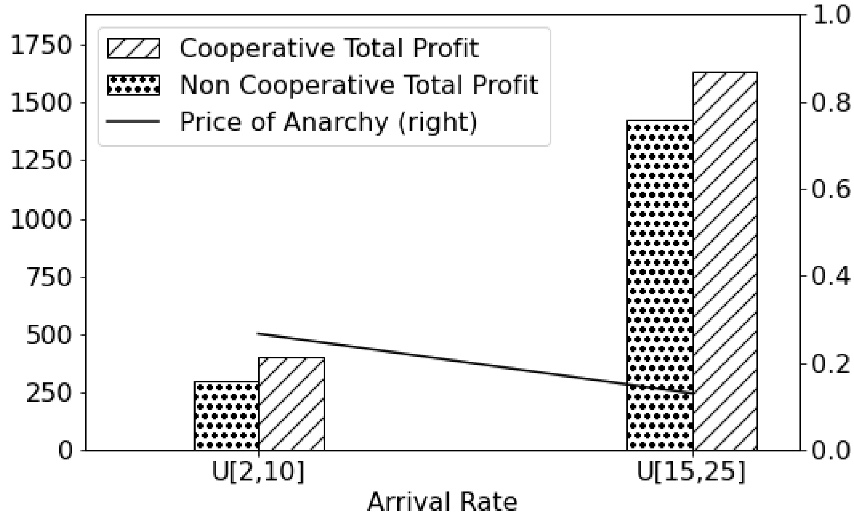

Figure 6.15: Change in total profit and price of anarchy under cooperative and non-cooperative games with respect to a change in λ

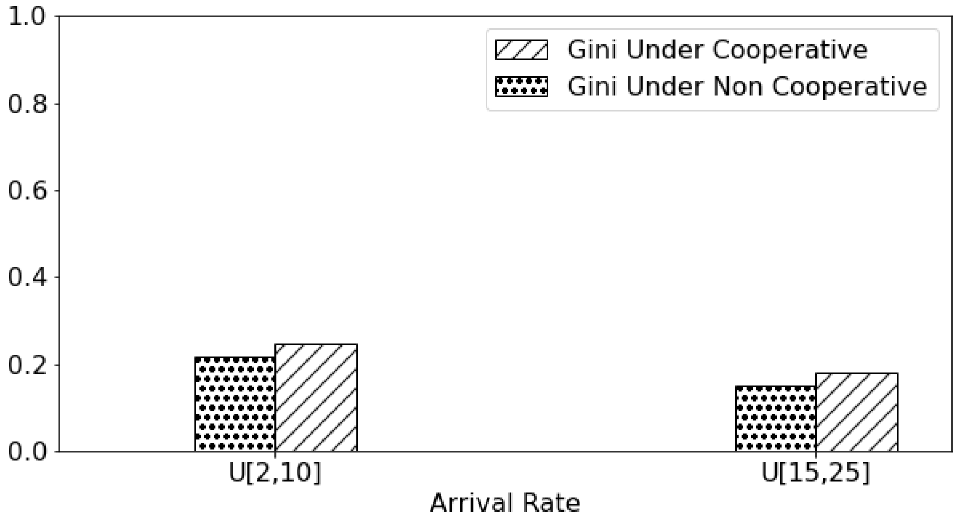

Figure 6.16: Change in Gini coefficient under cooperative and non-cooperative games with respect to a change in λ

6.6 Summary of Results

To sum up our findings in all sections, when number of shippers are increasing total profits made under both games increases and individual profits are slightly increasing. Profits under cooperative game are higher so there is price of anarchy emerged from selfish behavior of shippers. The gini coefficient fluctuates with increase of N. As N increases, noncooperative game seems to preferable because the existence of free shippers make profits which is more close to equal allocation of profits.

The effect of transportation cost change can be seen as straightforward. The increase of A decreases total and average individual profits. The price of anarchy consistently increasing, so selfish behavior creates more inefficiency and profit loss. The effect of free shippers can also be seen here. The Gini coefficient under non-cooperative games is lower than cooperative games. Shippers allocation seems more close to each other.

The effect of revenue change is also quite logical. Profits increase when per unit revenue of each shipper increases. Price of anarchy decreases with respect to an increase in revenue. This shows that shipper groups having more revenue do not face with high level inefficiency and they are more likely to attend selfishly.

The effect of waiting cost change affects average individual and total profits negatively. Price of anarchy increases with the increase of waiting cost. So, there becomes more efficiency loss. Due to the allocation rule of cooperative game, which is partial shippers takes no profits, the differences in allocated profit are higher in cooperative game and the gini coefficient of non-cooperative game is lower than the cooperative one.

The effect of arrival rate change affects average individual and total profits positively. Price of anarchy decreases with respect to a increase of arrival rate. This shows that shipper groups having more revenue do not face with high level inefficiency and they are more likely to attend selfish behavior similar to revenue increase. The increase of arrival rate slightly decreases the gini coefficient which means the difference of allocated profits of shippers slightly decreases. The increase of arrival rate slightly decreases the Gini coefficient which means the difference of allocated profits of shippers slightly decreases. There is not a significant difference of the Gini coefficient between non-cooperative and cooperative games.

CHAPTER 7

CONCLUSIONS AND FUTURE WORK

In this study, we have shown that shippers can participate in joint-shipment with different active operating times. Firstly, we have started with the two shipper setting. For that, we have found the optimal operating time for each shipper, the optimal joint-shipment cycle time, and the maximum level of total profit that can be made. We have verified the conditions needed for doing profitable shipments. joint-shipments for each combination pair of self-sufficient and not self-sufficient shippers. Then, we generalized our findings on s shippers setting. We have described the total profit function and developed an algorithm for finding the region of T^* and determining full time and partial time shippers among s shippers. After all structural findings, we have described cooperative and non-cooperative games to study on allocation of total profit made by joint-shipments.

In the cooperative game, we have proposed our allocation scheme: Full time shippers take the same profit as their profit from the individual shipment case with transportation cost, A^i and partial time shippers do not take any profit from the coalition. It has proven that that allocation scheme satisfy the conditions of monotonicity, superadditivity, convexity and the core of the game exists. In the non-cooperative game, we have expressed the best response function and characterized the Nash Equilibria. It has found that the self-sufficient shippers who have same and smallest optimal cycle time of individual shipment will contribute an amount that makes the joint-shipment cycle time equals to their optimal cycle time of individual shipments. The remaining shippers will become free-rider and they will not contribute.

Shippers having different characteristics should prefer doing joint-shipment to decrease total transportation costs in a deterministic uncapacitated environment. if they act selfishly, instead of being in collaboration for the allocation of total profits, an efficiency loss occurs, which we have pointed out as the price of anarchy. It is calculated by the profit difference under cooperative game and non-cooperative game over the profit made by cooperative game. We made numerical analysis to quantify price of anarchy on average individual profits and total profits. We also check Gini coefficients of profit allocation schemes of cooperative and non-cooperative games.

We find that change in number of shippers, revenue and arrival rate affect the average individual and total profits positively. On the contrary, change in waiting cost and transportation cost loses decreases total and individual profits. The price of anarchy increases with cost increasing activities and decreases when the shippers become more eligible to make shipments individually. The Gini coefficient is not strongly correlated with price of anarchy but the free shippers in non-cooperative games and the partial shippers who do not take profits in cooperative games affects the Gini coefficient significantly.

There can be several potential extensions to our studying setting. There can be different preferable transportation modes having limited capacity and corresponding different A values. Choosing the optimal transportation mode considering capacity level and transportation cost will be another decision variable for shippers beside of T and p values for total profit function. Also, stochastic arrival rates of shipment demands can be studied. Another exciting extension could be multiple origin-destination pairs and considering travel times to consolidate shipments. When this addition is done, the transportation cost, A, may vary with corresponding distances, and the allocation of the profit may change accordingly.

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